Cultivate Your Creativity By Thinking Outside Of The Box

Carmen Kissoon-Ragoonanan
Beeber Middle School

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Overview

Research has shown that students in Japan are outperforming American students in Mathematics. In a 1987 study based on a math test given to 13-year-old students in 18 nations by the International Association for the Evaluation of Educational Achievement (IEA), students in Japan scored the highest. The United States ranked 14th. (Time to Close the Math Gap, fall 1992). The conclusion of this research demonstrated that the underachievement of students in the United States is a result of lack of time spent on math study. However, the influential work from this study opened the eyes of educators in several countries to concentrate on schools' math curriculum to close the gap.

On the other hand, suburban school districts students have outperformed students in the urban school districts. Some educators will argue that the suburban school districts have the availability of more extravagant monetary funds and are able to purchase more supplies. Some of my colleagues agree that teachers need to enhance the schools' curriculum by introducing inquiry -based learning into the classroom on a daily basis. As a result inquiry driven teachers apply the problem solving strategies in the classroom and help students to develop logical deductive thinking and use this as a tool for daily applications. According to Howard Gardner, inquiry based experiences tap into multiple intelligences- meaning many different learning styles are engaged. He also stated that inquiry based learning is education for the future.

In order to have students thinking outside of the box, this unit will include brain teasers, and problems that are required to use problem solving strategies such as making tables, making organized lists, looking for patterns, guess and check, drawing pictures or graphs, working backwards or solving simpler problems.

This unit is written for students in the 6th and 7th grade. These students have ninety minutes block periods for mathematics on a daily basis. I intend to incorporate this unit with the Mathematics in Context (MIC) unit "Comparing Quantities" which will be taught later in the series. In order to prepare this unit, I will have to apply the cognitive effect of problem solving on students and employ the National Council of Teachers for

Mathematics approach to problem solving. I intend to use various problem- solving strategies and engage students to apply these to think outside of the box. Problems included in this unit will be discrete math where students can use various strategies to figure out the advantages and disadvantages of each before choosing an approach. Ultimately, this unit will enable students to find the solution to a problem and become critical thinkers.

The following schedules are suggested for incorporating this unit into the schools' curriculum. Teachers can adapt these schedules to fit into their teaching block or curriculum. 1) Monday- 25 minutes, Tuesday, Wednesday, Thursday and Friday-20 minutes each. 2) Monday and Tuesday-25 minutes each with problem solving assigned as independent work and students working cooperatively. Brainteasers can be used as warm up or independent seat work.

Rationale:

Certification in mathematics is a challenge to some teachers. On the other hand, a teacher who is math-certified is more marketable. It can also be argued that some of our colleagues need to adapt their teaching strategies and have students investigate and explore numbers instead of just using formulas. More teachers need to adapt their methods of teaching in the classroom and have students explore varied problem solving strategies. Approximately 75% of my students are performing at basic or below basic in mathematics. Having students apply multiple problem solving strategies will enhance the schools' curriculum with the use of deductive thinking. This unit will help to motivate and improve students' performance with problem solving, applying problem solving strategies, and selecting the most appropriate solution strategies to obtain optimum solutions to problems and to help to close the math gap. In order for students to think outside of the "box", it is imperative to teach problem solving through extensive time, spread throughout the week, and on a daily basis throughout the curriculum. This will help to develop critical thinking skills, enhance the schools' curriculum, and provide some fun for students. This unit will also be integrated with writing and communication.

Background

What is problem solving? Problem solving can be defined as engaging in a task for which the method of solution to the task is unknown to students in advance. This task is challenging and can present students with difficulty in obtaining a solution. When the steps in the solution to a problem are clear, then this is no longer a problem but an exercise. Problem solving is inquiry-based learning where teachers in the classroom assist students to develop mathematical ideas through creating, exploring, teaching and verifying solutions. According to NCTM, good problem solvers have a "mathematical disposition." They analyze situations carefully in mathematical terms. (Principles and Standards for School Mathematics page1).

Specific characteristics of a problem solving approach include:
- Interactions between students/students and teacher/students (Van Zoest et al; 1994)

- Mathematical dialogue and consensus between students (Van Zoest et al 1994)
- Teachers providing just enough information to establish background/ intent of the problem, and students clarifying, interpreting, and attempting to construct one or more solution processes (Cobb et al 1991)
- Teachers accepting right/wrong answers in a non-evaluative way (Cobb et al 1991)
- Teachers guiding, coaching, asking insightful questions and sharing in the process of solving problems (Lester et al; 1994)
- Teachers knowing when to intervene, and when to step back and let the pupils make their own way (Lester et al; 1994) (Mathematics Through Problem Solving, Taplin, page1)

What is Inquiry-Based Learning (IBL)?
IBL is a form of teaching which involves the teacher as 'a guide on the side' rather than a 'sage on stage.' This simply means that the students are guiding their discovery, formulating their questions, and figuring out the solution to their questions. By doing this students are invested in their learning and are motivated to get results. (Inquiry Based Learning, fall 2001)

An old adage states: "Tell me and I forget, show me and I remember, involve me and I understand". Inquiry Based learning is based on the last part of this statement. According to Joe Exline, inquiry is involvement that leads to understanding.

Howard Gardner stated that these are the characteristics of an inquiry-based learning:

1) Using critical thinking skills (making guesses, connecting data and forming explanations, testing the explanations.)
2) Observing surroundings, happenings, (using visual clues, listening, counting) sketch and record data.
3) Collaborate with others. To summarize, inquiry based experiences tap into multiple intelligences- meaning many different learning styles are to be engaged.

According to the National Council for the Teachers of Mathematics (NCTM) the goals for problem solving are:

1) Students building new mathematical knowledge through problem solving.

2) Students solving problems that arise in mathematics and other contexts.

3) Students applying and adapting a variety of appropriate strategies to solve problems.

4) Students monitoring and reflecting on the process of mathematical problem solving. (Principles and Standards for School Mathematics NCTM).

Based on research, it is obvious that problem solving plays an important role to students and society. Whenever students develop problem-solving skills, this can become motivational for them. It is difficult to teach problem solving skills without context. Using the inquiry- based approach can become advantageous for students. They will

learn new concepts or skills, or this can reinforce the skills they already know and enhance logical reasoning.

Further exploring the National Council of Teachers in Mathematics standards and their findings on the effect of problem solving on students thinking critically is essential for this unit. Furthermore, the standards are connected to local, state and national standards.

Objectives

The objectives of this problem-solving unit include the following:

- To develop computational fluency that is accurate and efficient.
- To solve problems connected to the National Council of Teachers in Mathematics standards (NCTM)
- To build new mathematical knowledge through problem solving.
- To apply and adapt problem solving strategies such as draw a diagram, look for a pattern, solve a simpler problem, make an organized list or table, Estimation-through guessing and checking, work backwards, use logical reasoning, write equations or ratios.
- To think outside of the box and apply problem solving strategies to life experiences.
- To explore and justify solutions.
- To share all work and write a complete explanation of strategies used to solve the problem.
- To write word problem using problem solving strategies and have other groups find the solutions to these problems.

Strategies

The following problem solving strategies will be incorporated into students' inquiry learning:

The Four Square Approach

Using a graphic organizer can enhance a lesson. For example, vocabulary terms like average, median or mode are important mathematical concepts that students need to be able to demonstrate they know in multiple forms. Using Frayers' model of the four

square approach is one of the suggestions for building vocabulary. In the first square, students will write the definition of the word, second square, they will give a non example of the word, third square, they will give an example of the word, and in the final square they will draw a picture of the word or write a formula.

K-W-L- What you already know, what you Want to Know and What You Learned

Using a K-W-L graphic organizer is a good idea of activating students' prior knowledge of a problem-solving situation. This is to know what they know about a particular problem. This will also stimulate their thinking of things they want to know about this particular problem. The final column can be filled out at the end and this will summarize what they have learned after solving the problem. Each group can share what they have learned with the class. This will allow students to see a specific problem from a different perspective and looking at various solution strategies.

Draw a picture or Diagram

A vast majority of my students are visual learners and this is often used in my classroom. The use of charts, tables, flow charts and manipulative are strongly suggested where applicable. Manipulative can be used to model or demonstrate problem situations and solutions. Display a list of "Magic Words" in the classroom. This will assist students to explain their reason for choosing a particular solution strategy. Examples of magic words include the following: (to get, to see, to find, to show, to figure out, because, since)

Constructive Response

Standardized testing consists of constructive response questions. The Philadelphia School District required that teachers should use Tag it 3 strategy to teach constructive response questions. Students find this graphic organizer helpful to use. Students are required to use a part of the math problem into an opening statement, and give strategies or calculations used to solve the problem.

T- Charts

T-Charts are suggested for solving open- ended problems. This is a two - column chart that is used to solve the problem in the first column and write a step – by - step explanation in the second column. This will help students to describe each step in the solution. These are ideal for showing the solutions for constructed response questions. Constructed response questions are short answer questions that are open ended. The purpose of these questions is for students to apply their cognitive skills and incorporate this with the content knowledge that was taught. A scoring rubric must be developed for this. These questions can be used for both formal and informal assessment because they assess higher level thinking skills and they provide the opportunity for students to show their understanding of content beyond the basic level.

The Hand

Another graphic organizer students can use to help them to respond to constructed response question is the hand. This is frequently incorporated into my lesson. The five fingers represent a step in problem solving. Starting from the little finger, these are the five steps to problem solve:

1) Underline the question

2) Circle the facts

3) Choose a strategy

4) Solve the problem

5) Does your answer make sense?

The listed strategies are essential in developing problem solving skills.

Make a chart
Guess and Check
Draw a picture
Make a model
Look for a pattern
Work backwards
Use logical Reasoning

The following problem solving strategies can be used to find the solution to **Grade Anxiety problem below.**

- **Guess and check**
- **Use a spreadsheet**
- **Work backwards**
- **Use logical Reasoning**

Problem: In Wilbur's math class, his grade is based on the average score of six tests, each worth 100 points. Wilbur always worries about his grade. He knows that his average on the first four tests is 88.5. What is the lowest average he could get on his next two tests and still average 90 overall?

Suggestions for students or groups that are struggling

Guess and check until you find four scores which average 88.5.
(Some possible numbers are 88,88, 89, 89 or 87, 88, 89, 90 or 88.5, 88.5, 88.5, 88.5)
Guess what the next two numbers will be. Find the average of these guessed scores.

Make the necessary adjustments and make more guesses until the average comes out to 90. Find the average of the two guesses and this will be the solution.

Make a chart or a spreadsheet to compute the averages as the students make their guesses. Fill in four scores with an average of 88.5. Enter the formula for computing the average of all six scores. Guess scores for tests 5 and 6 until the average is 90.

# 1	#2	#3	#4	#5 guess	#6 guess	AVERAGE
88	88	89	89	94	98	91.00
88	88	89	89	94	97	90.83

Another approach is to consider combining the sums of test scores, rather than averages. Wilbur wants an average of 90 points over six tests, what total number of points does he need? His average on four tests is 88.5 how many points does he have? How many more points does he need to make his goals? Split this over his next two tests.

Complete Solution

6 * 90 = 540 points needed
4 * 88.5 = 354 points
540 – 354 = 186 more points needed
186 divided by 6= 93 points on each of the next two tests.

Complete Solution

Wilbur needs an average of 93 on each of the next two tests. (Problem Of The Week A Fresh Approach to Problem Solving: page 22)

Problem

A student averages 72 on five different tests. If the lowest score is dropped, the average rises to 84. What is the lowest score?

Solution:

Using the meaning of average, the average of the five scores is given by

$$\text{Average} = \frac{T_1 + T_2 + T_3 + T_4 + T_5}{5}$$

The score to be removed can be any one of those shown. We will call the one removed T1 simply because this is the first one on the list.

T1 = 5 Average 5 – (T2 + T3 + T4 + T5)

This equation requests the sum of the other four grades.
Again using the meaning of average, we have
T2 + T3 + T4 + T5 = 4 Average

Substitution gives
T1 = 5 Average – 4 Average

It is given that Average 5 = 72 and Average 4= 84

Substitution results in

T1 = 5 (72) – 4 (84)

From which

T1 = 24

The score removed to change the average from 72 to 84 is 24.
(Problem Solving Using Mathematics)

Explanation

This question can be further extended to an open -ended question. Students can write their explanation to their solution. A classroom teacher can use this as a model to solve a wide variety of everyday problems. Things that can be changed include the words, symbol, numbers, and the ideas.

It is clear that individual score cannot be found from the average, however, the sum of all the scores can be found. The average of all five tests is 72. The sum of all five tests is 72 * 5 = 360. We were told that with the lowest score dropped, the average rises to 84, so the sum of the four other scores (the higher ones) is 4 * 84 = 336.
The difference between the sum of all five scores and the sum of the 4 highest scores will be the lowest score; that is 360 - 336 = 24.

Working Collaboratively

The activities in this unit will be completed in small groups. Group size will be four students placed in heterogeneous groups. In order for this model to be successful, teachers should have excellent classroom management skills. Working collaboratively to solve problems is an important step for students to communicate and explain their solutions. Teachers should ensure that students are comfortable in their groups so that they can build a sense of community and develop positive social interaction, as well as team building. They should feel free to express their ideas openly and honestly. In the beginning, teachers should be selective with problems allowing students to cooperatively

create and use various representations such as to organize, record, select, apply, present, and explain their thinking about mathematical solutions.

When working collaboratively, teachers can start students with simple problems and introduce the three –step approach to problem solving. This three - step approach includes the following:

Step 1: Identify the request
What does the problem ask for?

Step 2: Respond to the request
"How would I find out?"

Step 3: Generate the result
" What does the result tell me?"

Teachers will point out to students that this is a simple three step approach that is based on request and response. This approach can be used in solving the problems is the suggested classroom activities / lesson plans.

Classroom Activities/Lesson Plans
Lesson 1 (Duration of time 20 – 25 minutes)

Ticket Trouble

When Mr. J. Raff took his two children to the zoo, their tickets cost $8.75 all together. The next family in line was Ms. Ella Fant with grandpa and the five Fant kids. Their tickets cost a total of $19.75. How much does the zoo charge for each adult and each child?

Students will read the problem, and use hi-liters to underline important information that is given. They will then figure out what the question is asking for by using the three step approach and decide on one of the problem solving strategies.

These are the steps to be used:

1) Identify the request
"What does the problem ask for?"

2) Respond to the request
"How would I find out?"

3) Generate the result
"What does the result tell me?"

Teachers will suggest possible strategies such as

- Guess and check
- Make a table
- Look for a pattern
- Use a spread sheet

Suggestions for groups that are struggling

Try the guess and check strategy, for example by guessing a price for the adult ticket. Subtract this amount from Mr. Raff's total and divide what is left by two to find the price of a child's ticket. Use these prices to determine the Fant's family total. If it is not $19.75, make a new guess and try again.

Teachers will suggest to students that they need to keep track of their guesses in a chart as shown below.

Adult ticket price(guess):	3.00	
Raff total (given):		8.75
Subtract 1 Raff adult	-3.00	
Left over:		5.75
Divide for 2 Raff kids		2.875
Kid's ticket price:	2.875	
Fant adults (2):		6.00
Fant kids (5):		14.375
Fant total:		20.375

Extension Activity-

Teachers who have access to a computer lab can have students make this chart on a spreadsheet. Teachers will have to tell students to enter the appropriate formula to calculate the kids' ticket price based on the guess of an adult ticket. Enter formulas for determining the Raff's total and the Fants' total. Let the computer generate several lines of the chart and zero in on a solution.

Suggest to students that they can look for a pattern in the table. Each time that there is a dollar increase in an adult ticket, how is the Fant's total affected? Students will be able to look at this pattern and predict the ticket price.

Complete Solution
This is an example to the solution of the problem.

Adults ticket cost $4.25, and children's ticket cost $2.25.

Adult Ticket (guess)	Kid Ticket	Raff Total	Fant Total
$3.00	$2.88	$8.75	$20.38
$2.00	$3.38	$8.75	$20.88
$4.00	$2.38	$8.75	$19.88
$5.00	$1.88	$8.75	$19.38
$4.50	$2.12	$8.75	$19.62
$4.25	$2.25	$8.75	$19.75

Students should observe that whenever the adult ticket price drops, the Fant total increases. Students can share their strategies used and their solutions or use one of the graphic organizers such as the T-Chart to write a step-by -step explanation.
(Problem Of The Week - A Fresh Approach To Problem Solving)

Lesson 2
Duration of time (20 – 25 minutes)

Kyle had 36 books in his locker. Some were comic books, some were text -books, and the rest were sports magazines. The number of comics and sports magazines combined equals twice the number of textbooks. The number of textbooks and sports magazines combined equals three times the number of comic books. How many of each type of book were in Kyle's locker?

The following problem solving strategies are suggested to find the solution to the above problem.

- Guess and check
- Act it out
- Use logical reasoning
- Make a chart

The following suggestions are recommended if students are stuck.

Guess how many comics and sports magazine are there. Use these to determine how many textbooks there would be according to the first clue. See whether this gives a total of 36 books. Now check these values using the second clue. Adjust guesses based on information from each previous guess.

A chart can be used to keep track of guesses. Look for relationships that might lead to the correct values.

Comics	Sports Mag.	Text	Total	
10	12	11	33	
10	14	12	36	(12 + 14 = 3(10)

NO

8	16	12	36	(12 + 16 = 3(8) NO

Complete Solution

Comics	Sports Mag.	Text	Total	
10	12	11	33	
10	14	12	36	12 +14 = 3(10) NO
8	16	12	36	12 +16 = 3(8) NO
9	15	12	36	15 + 12 = 3 (9) YES

9 + 15 +12 = 36 YES
9 + 15 = 2(12) YES
15 + 12 = 3 (9) YES

(Problem of the Week – A Fresh Approach to Problem Solving)

This brain-teaser is suggested as an additional activity if students have completed the problem before time.

A street vendor sells two types of newspapers, one for 25 cents and the other for 40 cents. If in one day she sold 100 newspapers and took in exactly 28 dollars, how many of the 25 - cent newspaper did she sell?

Students can use the hand model to solve the above problem. The five fingers represent a step in this model. These steps are to underline the question, circle the facts, choose a strategy, solve the problem, and ask yourself if your answer make sense.

Students can use a T-Chart to show their solution. In the first column they will be required to show their work and in the second column they will write a step- by -step explanation to their solution.

Solution

Let x be the number of 25 cent newspaper and y be the number of 40 cent newspapers.

$x+ y = 100$

From the money data

$0.25 * x+ 0.40y = 28$

$y = \underline{28 – 0.25x}$

$$\frac{0.40}{}$$

Substitution gives

$$x+\frac{28 - 0.25\,x}{0.40} = 100$$

Solving for x gives
X = 80

It doesn't make any difference which equation is used first

$0.25x + 0.40y = 28$

$x + y = 100$

$y = 100 - x$

$0.25x + 0.40(100 - x) = 28$

$x = 80$

(Brain Teaser provided by Kaplan Education Centers)

Lesson 3
Duration (20 – 25 minutes)

This problem can be incorporated with the Mathematics in Context Program, and can be used with the unit "Comparing Quantities."

Compare and Exchange – Bartering
Present the following scenario to the class. (This information should be given to students in writing so that they can look for the math in the reading.)

There was a time when money didn't exist. People lived in small communities, grew their own crops, and raised cattle or sheep. What did they do if they needed something they didn't produce themselves? They traded some of the things they produced for the things they needed. This is called bartering or exchanging.

Paulo lives with his family in a small village. His family needs corn. He is going to the market with two sheep and one goat to exchange them for bags of corn. First, he meets Aaron, who said, "I only trade salt for chickens. I will give you one bag of salt for every two chickens."

"I don't have any chickens," thinks Paulo, so I cannot trade with Aaron.

Later he meets Sarkis who tells him, “I will give you two bags of corn for every three bags of salt.

“That doesn’t help me either.” Paulo thinks.

Then he meets Ranee. She will trade six chickens for every goat, and she says, “My sister, Nina is willing to give you six bags of salt for every sheep you have.”

Paulo is getting confused. What can he do? He has to come home with just bags of corn, not with a goat or sheep or chickens or salt.

1) Show what Paulo can do.

The following problem solving strategies are suggested:

- **Draw a picture or diagram**
- **Make a list**
- **Use** logical **reasoning**

Allow students to struggle with this bartering problem. Encourage them to develop their own problem solving strategies. The exchange from salt to corn may be the most difficult because the ratio of salt to corn is 3: 2.

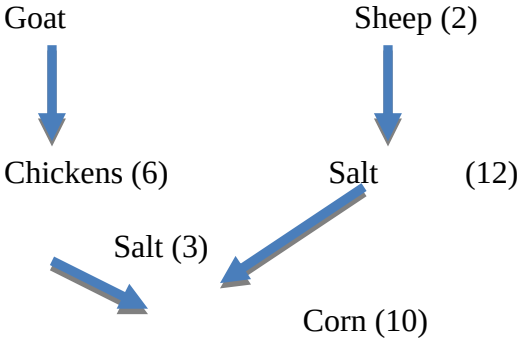
Allow each group to present their solution using the three step approach, the hand, or a T- Chart. This is an opportunity to use the four square approach to teach vocabulary. The new vocabulary word from this lesson is bartering. Students will define this in the first square, write the characteristics in the second square, give examples in the third square, and give non-examples in the fourth square.

Strategy 1 (Logical Reasoning)

Paulo should trade the 2 sheep for 12 bags of salt and the goat for 6 chickens. Then he should trade the 6 chickens for 3 bags of salt. Paulo will have a total of 15 bags of salt. He should trade the 15 bags of salt for 10 bags of corn and go home.

Strategy 2

Use a tree diagram



Strategy 4

Organize the solution according to the exchange for each merchant.

1st :	Nina	2 sheep	12 salt
2nd:	Ranee	1 goat	6 chickens
3 rd:	Aaron	6 chickens	3 salt
4th :	Sarkis	15 salt	10 corn

Writing Opportunity: Write a journal entry describing a personal experience in which something was bartered or traded in exchange for something else.

Share did you know information with the class.

Did You Know?

Bartering means trading goods and services instead of paying money for goods and services. Purchasing goods with money is more convenient than trading because money is easily carried. (Comparing Quantities Holt, Rinehart and Winston)

Extension Activity:

Students can write their own word problem. They can exchange these problems with other groups of students to solve.

If students are finished before time they can be assigned the following brain -teaser.

My towing rope was cut in half

And half was thrown away.

The other half was cut again

One- third along its way.

The longer part (ten meters length)

Is what I use today.

But how long was my towing rope

Before this cutting fray?

(A Professor Morris Problem)

Solution

The original length of the rope is what was kept and what was discarded.
Original length = used today + discarded

This equation first asked for used today.
This is given in the problem statement as used - today = 10meters.

The current requesting equation next asks about the amount discarded. From the problem statement this is ½ the original length plus 1/3 of half of the original length.

$$\text{Discarded} = \frac{\text{original} - \text{length}}{2} + \frac{1}{3}\,\frac{\text{original length}}{2}$$

Substituting these responses into the equation that requested the information provides

$$\text{Original length} = 10\text{m} + \left(\frac{\text{original} - \text{length}}{2} + \frac{1}{3}\,\frac{\text{original length}}{2}\right)$$

Solving this for original length

$$\text{Original} - \text{length} = 10\text{m} + \frac{2\ \text{original length}}{3}$$

10m 5m 15m

Extension: Students should be given the opportunity of writing their own math problem using a poetic format. Teachers can use this as a writing opportunity and make use of figures of speech and rhyming scheme. Students can share this with the class by exchanging papers and allowing other students or groups to apply the problem solving strategies to fin the solution.

(A Professor Morris Problem)

Lesson 4
Duration (20 -25 minutes)

Compare and Exchange – Bartering

Present the following scenario to the class. Delilah lives in a community where the people trade goods they produce for other things that they need. Delilah has some fish that she caught, and she wants to trade them for other food. Delilah hears that she can trade fish for melons. She wants more than just melons, so she decides to see what else is available. This is what she hears.

- For five fish you can get two melons.
- For four apples you can get one loaf of bread.
- For one melon you can get one ear of corn and two apples.
- For ten apples you can get four melons.

1) Rewrite or draw pictures of this information so that it is easier to use.

2) Use the statements above to make up two more statements about the exchanging of apples, melons, corn, fish, and bread.

3) Delilah says, “I can get 10 apples for 10 fish.” Is this true?

4) Is it also true that she can trade three fish for one loaf of bread? Explain why or why not.

5) Explain how Delilah can get some ears of corn.

The following problem solving strategies are suggested to figure out the solutions to the questions.

Draw pictures or symbols
Make a list
Logical reasoning

These problems assess students’ ability to use pictures, words, or symbols to solve problems. It is suggested that students share their strategies, describe their solution process and follow the step-by-step process of other students.

Problem 5 does not state the number of fish that Delilah has to trade for corn. Some students may trade for some corn while others may trade until they have only corn.

Solution

1. 5 fish = 2 melons
   4 apples = 1 bread
   1 melon = 1 corn + 2 apples
   10 apples = 4 melons

2) 5 fish = 2 melons
   1 fish = 1 apple
   1 fish = 2 corn
   4 fish = 1 bread
   2 melons = 5 apples
   1 melon = 5 corn
   8 melons = 5 bread
   1 apple = 2 corn

4 apples = 1 bread
8 corn = 1 bread

3) Yes. Students will come up with different explanation. Here is a sample solution.

Since 2 melons = 5 fish
Then 4 melons = 10 fish
Also 4 melons = 10 apples
So 10 fish = 10 apples.

4) No, it is not true. Explanations will vary. Here is a possible solution. Problem **8** shows that 1 fish = 1 apple, so 4 fish = 4 apples. Problem **6** shows that 4 apples = 1 bread, so 1 bread = 4 apples = 4 fish. So, three fish are not enough for one loaf of bread.

5) Students are going to have various explanations. Here is a sample explanation. Delilah can trade five fish for two melons. For the two melons she can get two ears of corn and four apples. Then she can exchange the four apples for eight ears of corn to get a total of 10 ears of corn.

Extension

Teachers can apply the exchange or substitution principle to money using nickels, dimes, quarters and dollars.

(Comparing Quantities – Holt, Rinehart and Winston)

Students who have completed the problem assigned before time can be assigned the following brainteaser. This can also be used as a homework assignment.

Brain Teaser

Parade of Legs

Mary looked out of her farmhouse window and saw a group of pigeons and cows passing by. She counted all the legs of the pigeons and cows and found that the total number of legs add up to 66. How many of each kind of animals (pigeons and cows) passed by her window if the total number of animals is 24?

Solution

Since pigeons have 2 legs and cows have 4 legs, the number of animals is related to the number of legs, 66, by

2 pigeons + 4 cows = 66

We will solve first for “pigeons.” The equation then requests information about the number of cows. It is given that the number of animals is 24 so the number of cows is described by

Cows = 24 – pigeons.

Substitution provides
2 pigeons + 4(24 – pigeons) = 66
Solving this for “pigeons” results in
-2 pigeons + 96 = 66
-2 pigeons = -30
pigeons = 15 so that cows =24 – 15 = 9
15 pigeons and 9 cows passed by Mary’s farmhouse window.

Teachers can start students with simple problem solving strategies using the three-step approach. They are to constantly remind students that they are to respond directly to what is asked for. This approach will lead to the solution of the problem. Starting a problem is the simplest, least time consuming, part of solving a problem.

Discussion can be based on the following. If all the animals were pigeons, the number of legs would be 48 but the actual total is 66 legs. The difference is 66 – 48 = 18 legs. When a pigeon is “changed” into a cow, the difference is 2 legs. Therefore, the number of pigeons that need to be “changed” into cows is 18 divided by 2 = 9.
Answer: 9 cows and 15 pigeons passed by Mary’s window.

Suggestion: Simplification of the solution to this problem can be obtained by avoiding the use of algebra.

The following problem solving strategies can be used to solve the Parade of Animals brain-teaser.

Make a table
Guess and check
Draw a picture

(This problem is adapted from Van De Walle, John (1994), Elementary School Mathematics (2nd Ed.), NY: Longman, pages 40- 41

Bibliography

Reading List

Demos, Glenda; and Linda Griffin. Problem of the Week. Instrumental Fair. TS Denison Grand Rapids, Michigan 49544

This book use several problems of the week to be incorporated in the schools' curriculum using problem solving strategies.

Holt, Rhinehart and Winston. Comparing Quantities. Copyright @ 2003 Encyclopedia. This book informally shows students how to use exchange strategies, substitution, combination charts, and notebook notation to solve equation like problems.

Problem Solving Using Mathematics. http//www.hawaii.edu/suremath/k4-12dir/changeGrade.html
This article shows the three steps problem solving method which are: identify the request, respond to the request, and generate the result.

Teacher Resources

Core Curriculum, School District of Philadelphia. Copyright@2003 by The School District of Philadelphia. The state standards were taken from the core curriculum.

Demos, Glenda; and Linda Griffin. Problem of the Week. Instrumental Fair. TS Denison Grand Rapids, Michigan 49544
This book use several problems of the week to be incorporated into the schools' curriculum using problem solving strategies.

Holt, Rinehart and Winston. Comparing Quantities. Copyright @2003 Encyclopedia. This book informally show students how to use exchange strategies, substitution, combination charts, and notebook notation to solve equation like problems.

Problem Solving in Mathematics
http://library.thinkquest.org/25459/learning/problem/#strategy
This article defines a mathematical problem, state the problem solving strategies and the four phases in solving a problem.

Rusczyk, Richard. What is Problem Solving? Reprinted from www.artofproblemsolving.com This article portrays how the that problem solvers developed in math transferred into other subject areas.

Problem Solving Using Mathematics. http://www.hawaii.edu/suremath/k4-12dir/changeGrade.html
This article shows the three steps problem solving method which are: identify the request, respond to the request, and generate the result.

Teacher Resources

Demoss, Glenda and Linda Griffin. Problem of the week. Instructional Fair. Ts Denison Grand Rapids, Michigan 49544
This book provides examples of problems of the week and their solutions.

Holt, Rinehart and Winston. Comparing Quantities. Copyright c 2003 Encyclopedia Britannica, Inc.
This unit is a part of the schools' curriculum and the concept of using some of the problem solving strategies will be taken from this.

Problem Solving in Mathematics. http://www.hawaii.edu/suremath/k4-12dir/changeGrade.html The four phases in problem solving will be taken from this as well as the problem solving strategies. Brain Teasers will be taken from this.

Mathematical Problem Solving Strategies. http://www.qerhs.k12.nf.ca/projects/math-problems/intro.html
The following steps in problem solving will be incorporated into my lessons: Read the problem, note key words, explore, select a strategy, solve and look back.

Overview: Standards for School Mathematics: Prekindergarten through grade 12. http://standards.nctm.org/document/chapter3/index.htm
The problem solving standards (NCTM) will be taken from this and incorporated with the school district standard.

Student Resources

Holt, Rinehart and Winston. Expressions and Formulas. Copyright c 2003Encyclopedia Britannica. Inc.
This is a part of the schools' curriculum and problems from this unit will be used in the classroom.

Demoss, Glenda and Linda Griffin. Problem of the Week. Instructional Fair. TS Denison. Grand Rapids, Michigan. Several problems will be taken from this source and assigned to students.

Appendix 1: Standards

The School District of Philadelphia designed a core curriculum, which is aligned to the Pennsylvania State Academic Standards. These Standards incorporate lessons in which students are required knowledge of Mathematical reasoning and connections and Mathematical problem solving and communication. Lessons designed in this unit will address the following mathematics standards:

2.4 D Mathematical Reasoning and Connections- Distinguish between relevant and irrelevant information in a mathematical problem by determining important information to solve a problem.

2.5 A Mathematical Problem Solving and Communication- Students will be able to invent, select, and justify the appropriate methods, materials and strategies to solve

problems. They will use knowledge that was learned previously to select one of the problem solving strategies to solve word problems.

2.5 B Mathematical Problem Solving and Communication- Students will be able to develop and use suitable mathematical terminologies, vocabulary, and symbols to logically explain their solutions to the problems assigned.

2.5 C Mathematical Problem Solving and communication- Students will be able to make connections to other problems and to generalize the solutions to problems to other concepts.

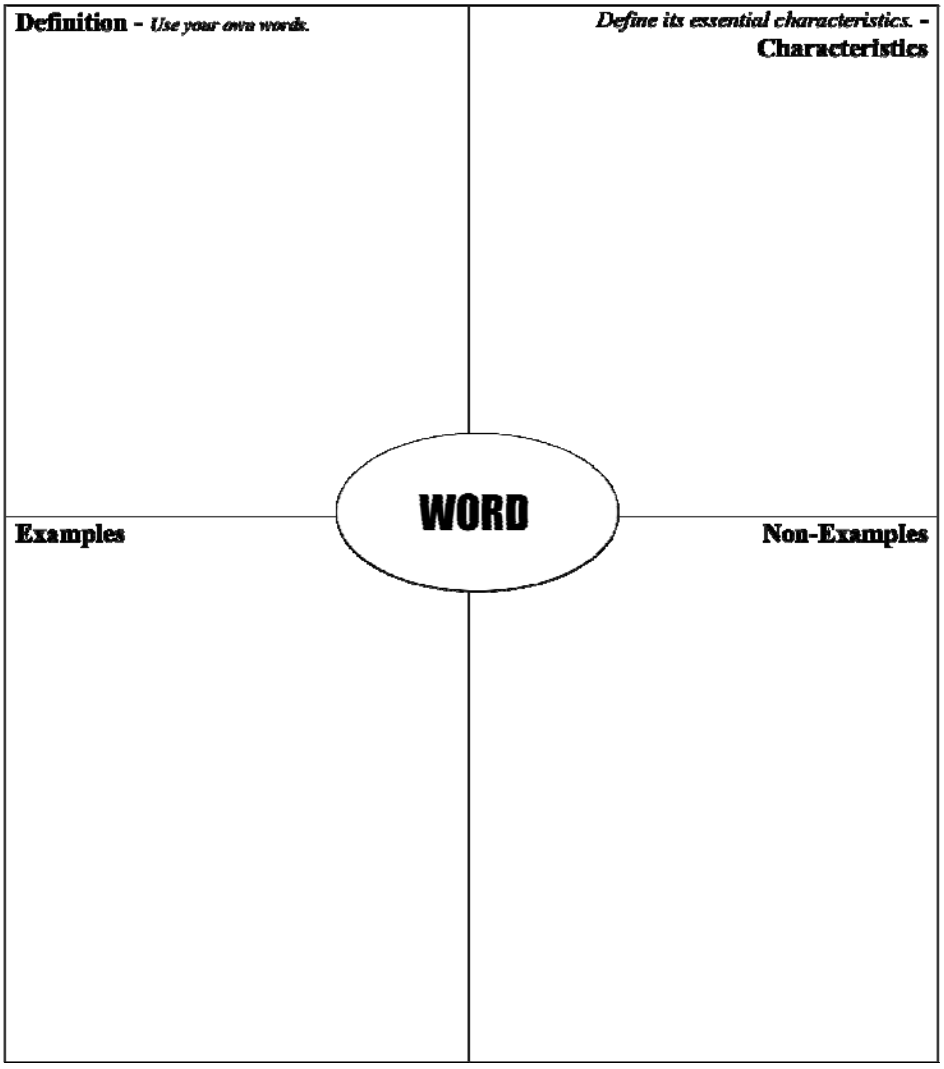
2.5 D Mathematical Problem Solving and Communication- Students will be able to choose the appropriate algorithm, select appropriate tools, and justify choices made to solve a problem.

2.5 E-Mathematical Problem Solving and Communication- Students will be able to develop strategies that show support for their solution and use simpler problems, pictures, diagrams, and drawings to support problem solving.

Appendix 2

Frayer Model Template

NAME _____ Date _____ Pd. _____



Appendix 3

Thinking Outside the Box T chart

Name:_____ Date:___

Solution	Explanation