Inter-disciplinary Exploration of Order: Looking Through the Commutative Property Lens

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Overview

The function of secondary school is to nurture the minds of adolescents, and educators aim to prepare youth for success in the future. One means to attain these goals is to provide students with an array of knowledge. However, the educational system cultivates a misleading sense of compartmentalization of subject areas. From the roster of separate courses to standardarized tests separated into content-based sections, students are raised in a learning environment that segregates material. By contrast, the integration of mathematics, science, literature, and other disciplines dominate the "real world". The audience for this curriculum unit is primarily 9th grade students in my Algebra 1 classroom, a vocational high school in inner city Philadelphia. Additionally, the intention of the series of lessons is to lessen the isolation between subjects and develop critical thinking skills and a love for learning and inquiry through problem solving. The curriculum's structure is seven segments: overview, rationale, mathematical background, strategies, classroom activities, annotated bibliography/resources, and appendix.

Rationale

In order for students to be proficient on the standardized exams such as PSSA, SAT, and ACT, they need to be able to solve a problem not encountered previously and apply the knowledge they have learned in various courses. The demand for this ability to tackle problems using application of knowledge pervades life beyond the school walls. All employers require their staff to utilize critical thinking skills to provide solutions to presented problems. Therefore, in order to prepare students for success in the years beyond high school, instruction needs to develop critical thinking skills in students.

Unfortunately, to the contrary, some of the present educational system promotes memorization and regurgitation of information. Students are not taught how to “think” but rather how to spit back out the information for the short period of time required to pass the test. This curriculum unit is designed to extend critical thinking, context, and cognition through creating connections between disciplines. Through the utilization of the mathematical concept of the Commutative Property as a springboard, the curriculum leads students to discuss the effects on change of order in concepts found in literature and science.

To overcome the impression that subjects are compartmentalized, the curriculum incorporates various activities and resources, which stress the importance and effectiveness of inter-disciplinary knowledge. Furthermore, the lessons include problems from several content areas and require knowledge from multiple disciplines to determine the solution. Therefore, students attain higher-level learning in juxtaposition with appreciation of the need to mastery and integrate various subjects to most effectively solve problems.

Unfortunately, the love of learning and investigation is becoming less prominent in our classrooms. Class is teacher-centered with lectured lessons in a room void of interaction and discussion. A more engaging curriculum would foster a love for learning and exploration by providing students with the tools to tackle the intimidating unknown. Often, students are presented with a problem they have not encountered before, and immediately shut down because they feel defeated. By incorporating and applying problem solving strategies taught and utilized through the curriculum, students will be able to feel confident and comfortable with exploring and discovering the unprecedented. Additionally, many students struggle with mathematics and therefore the content area carries a stigma of being boring, useless, and difficult. With the requirements of creating “real world” examples and non-examples of commutativity and solving “real-world” problems across the disciplines, will exemplify for students the existence of math in the world outside the four walls of the classroom. Furthermore, students will be less intimidated and more at ease with doing mathematics.

Another weakness for young adults is communication skills. I have often found students present the correct answer, however, they struggle with conveying the rationale behind their reasoning. The ability to communicate one’s thought process and reasoning behind one’s problem solving strategies is essential for success. Hence, one of the focuses of the curriculum is on students’ explanations and reasoning behind their choices. Also, at the conclusion of the curriculum, students in groups will facilitate a “mini lesson” in the form of a presentation with their own cross-curricular connections as well as justification behind strategies and step-by-step explanations of thought processes.

Additionally, the curriculum promotes higher level learning through expanding students' vocabulary, incorporating articles and literature, and familiarizing students with technological tools.

Mathematical Background

Traditionally, in Algebra class, the Commutative Property is taught to students in the unit focused on operations and real numbers. It is introduced as two separate properties, one holding for addition and one for multiplication, using words and variable notation. The independent practice for the Commutative Property provided in a textbook usually consists of identification of the property and numerical application. In the Holt, Rinehart, and Winston textbook for Algebra 1, the textbook mandated for Philadelphia public high schools math classrooms, introduces the Commutative Property as follows:

Compare the sums in columns A and B. How are they alike and how are they different? Compare the products in columns C and D. How are they alike and how are they different?

Sums

Column A	Column B
56 + 11 = 67	11 + 56 = 67
28 + 32 = 60	32 + 58 = 60
13 + 87 = 100	87 + 13 = 100

Products

Column C	Column D
6 * 11 = 66	11 * 6 = 66
8 * 9 = 72	9 * 8 = 72
10 * 54 = 540	54 * 10 = 540

Columns A and B illustrate the **Commutative Property of Addition**. Changing the order of the numbers in an addition expression does not change the sum.

Columns C and D illustrate the **Commutative Property of Multiplication**. Changing the order of the numbers in a multiplication expression does not change the product.

Changing the order of two terms in an expression is known as *commuting the terms*.

Commutative Property of Addition
For all real numbers *a* and *b*:
$a + b = b + a$

Commutative Property of Multiplication
For all real numbers *a* and *b*:
$a * b = b * a$

Two examples of the exercises provided to practice the concept of the Commutative Property are below:

1. Use the Commutative Property to find the sum: (27 + 98) + 73
2. Name the property illustrated: 32 + 17 = 17 + 32.

The textbook provides a brief investigation, followed by clarification and general notation. However, the students are directly given an explanation and the presentation is dry and straightforward. Furthermore, the questions given to practice the skill are simplistic and lack depth. The concept of a change in order and investigating the resulting effect is a complicated and intriguing concept. Therefore, providing opportunities for students to explore this idea will further their understanding and critical thinking and problem skills.

Palindromes are touched upon in the English curriculum in the introduction of literary terms/devices. Traditionally, a lesson or two is devoted to learning palindromes. Students are presented with the definition and often identify palindromes and discuss its significance in poetry. The segment of palindromes in this curriculum support the curriculum's overall goal of revealing to students the link between disciplines through the investigation of palindromes in math as well as English. Therefore, this curriculum offers students the opportunity to explore and compare the concept of palindrome in more than one subject area. Furthermore, often famous palindromes, such as "Madam, I'm Adam", "Able was I ere I saw Elba", and "A man, a plan, a canal: Panama", are often research assignments for students to find their significance in history.

Equilibrium systems are introduced in the physical science textbook written by Holt in the chapter entitled chemical reactions. While students are first taught about chemical reactions as going to completion, the concept of reversible equations is introduced along with equilibrium. The text provides the following example:

For example, carbonated drinks, such as soda, contain carbon dioxide. These drinks manufactured by dissolving carbon dioxide in water under pressure. To keep the carbon dioxide dissolved, you need to maintain the pressure by keeping the top on the bottle. Opening soda allows the pressure to decrease. When this happens, some of the carbon dioxide comes out of the solution, and you see a stream of carbon dioxide bubbles. This carbon dioxide change is reversible.

CO_2 (gas above liquid) $\rightleftharpoons$ CO_2 (gas dissolved in liquid)

Furthermore, students learn the proper notation for writing chemical equations, in particular, $\rightleftharpoons$ indicates a reversible change and $\rightarrow$ indicates a change that change goes in one direction – toward completion (products). In addition, students learn about Le Chatelier's Principle as a general rule that describes the behavior of equilibrium systems, if a change is made to a system in chemical equilibrium, the equilibrium, shifts to oppose the change until a new equilibrium is reached. The students investigate the effects of temperature, pressure, and concentration on a reaction. Examples directly from the textbook, and therefore curriculum, of types of problem directly connected to the previous above content are below:

1. Describe what can happen to the reaction rate of a system that is heated and cooled

2. Analyze the error in reasoning in the following situation: A person claims that because the overall amounts of reactants and products don't change, a reaction must have stopped.

3. Consider the decomposition of solid calcium carbonate to solid calcium oxide and carbon dioxide gas: Heat + CaCO3 -> CaO + CO2 (gas). What conditions of temperature and pressure would you choose to get the most decomposition of CaCO3? Explain.

I also choose to include chemical reactions, specifically, equilibrium, as another expansion of the Commutative Property. An equilibrium equation centers on the principle of "forwards is the same as backwards". This principle is also the heart of a palindrome. Therefore, the curriculum creates the links for students to connect to reach the overall objective of the unit. Furthermore, similar to math, many students are intimidated and fear science. Therefore, the curriculum introduces a potentially complex and bewildering concept by a more understandable approach. When the students build the connections between the ideas and disciplines as they progress through the curriculum, students learn the concepts are not all separate and complicated entities but rather a one compartment with each content piece enhancing the overall structure.

Furthermore, the curriculum develops students' critical thinking skills. In particular, there is a lesson focused on problem solving skills. Learning effective problem solving skills is a necessary life long competence for success. No matter which disciplines the student studies in high school/post secondary school and profession chooses in adulthood, reasoning and inquiry are vital for achievement. This curriculum teaches students some basic problem solving skills, how to use them as efficient tools, then applies them to tackle possibility difficult problems. The curriculum enables students to understand these strategies to the degree where they can apply them to problems they encounter dealing with a variety of disciplines.

Strategies

The curriculum is designed with the 9th grade Algebra 1 student in mind. Even though most effective in its entirety, the unit can be implemented in the classroom to the extent chosen by the teacher. The curriculum of the unit is structured in such a way that lessons can be utilized individually, as a core lesson or as an enrichment supplement. My purpose in designing the unit is to foster critical thinking in high school students. Based on my experience as a high school teacher in an urban school district, where the students are weakest is in area of critical thinking, students struggle with analyzing and evaluating information to formulate a conclusion. This struggle permeates into all subject areas. Based on my school's data from 11th graders scores on the state test called the Pennsylvania System of School Assessment (PSSA), it is clear students' weakness are in reading comprehension and constructed response, two areas which require strong critical thinking skills. Particularly in math, when students are given a problem out of context of the chapter, they struggle with how to tackle the problem. The students do not often know how to begin and which math concepts are applicable to the particular problem. Therefore, the aim of this curriculum is to have high school students formulate an answer to a complex essential question through learning and applying problem solving strategies.

In the curriculum, students will aim to answer the question, when and how is order important? Each lesson guides and provides the students with the necessary tools to be able to fully respond to the query. Since 21st century skills are essential for success in present day society, the curriculum incorporates technological resources through interactive Promethean board, websites, videos, Web Quests, virtual manipulatives, and applications (Inspiration, PowerPoint, Podcasts, etc). The curriculum provides structured instruction for both the teacher and student with seven-step lesson plans. Furthermore, the lessons break down the distinct lines between content areas by connecting three subjects, math, English, and chemistry, along with problem solving strategies through students by presenting "mini lessons". Rubrics, deadlines, and a variety of activities will be provided for the instructor.

At the start of the curriculum, Students will be introduced to the concept of commutativity. They will explore this idea through "real world" examples and counter-examples. Additionally, students will develop their own examples and non-examples of commutativity and provide written explanations for their choices. To supplement the problems introduced to students throughout the curriculum, a lesson is given on several problem-solving strategies. Students will learn when each strategy is used most effectively and appropriately.

The concept of commutativity will be linked to mathematics with the Commutative Property. Students will master proper mathematical notation, use of manipulatives (tangible and virtual) and numbers. The instruction will then expand on the concept of the Commutative Property through the presentation of traditional math problems involving numbers. The problems will be a collection of operational and word problems. For each problem, students will be required to explain their thought process to ensure students transfer their thoughts to paper using proper words and appropriate vocabulary. In addition, students will undergo an investigation. An investigation is student-centered activity, which involves a proposed question at the start of the lesson. The students are guided to construct a response to the question through activities and "leading-up" questions.

This particular investigation proposes the question: Does commutativity hold for each operation? It will explore the effects of changing order over the basic operations of addition, subtraction, multiplication, and division and students will be asked to explain the reasoning behind their answer, using evidence from the activity. The concept of order will be extended through the connection to palindromes in English and equilibrium chemical equations in physical science/chemistry.

To further foster students' critical thinking skills, the curriculum will conclude with a cooperative learning activity. Each group will be given a complication of random problems involving a question related to order. The problems will increase in difficulty. Students will be required to apply the knowledge attained from the instruction to solve the problems. Furthermore, they are to identify and explain the problem solving strategy or strategies used to find the solution to the problem and include a step-by-step description of the thought process.

Classroom Activities

Standard lesson

The following lesson plans are designed for a 45-minute period and take the form of the well-known Dr. Madeleine Hunter's seven-step lesson plan (http://www.humboldt.edu/~tha1/hunter-eei.html). The entire curriculum's duration is 12 classes (45 minute periods) and therefore around two full school weeks and 2 days. If you teach block scheduling, you can implement more than one lesson on a given day, shortening the duration. In addition, the curriculum is designed for the lessons to be used individually or groups, not just as a whole 12 pack. Therefore, if the teacher does not have the flexibility in his or her curriculum to incorporate the entire set of lessons, he or she can choose lessons to supplement his or her instruction. Moreover, the activities within the lesson were designed for the option to be used individually as well.

The lessons mirror the structure of instruction in my classroom during the year. On the white board for every lesson is a listing of the objectives and standards. Therefore, they are visible and referable to all students throughout the lesson. Furthermore, during instruction, I will reference the objectives and/or standards to fortify the lesson. It is important in all lessons to incorporate checking for understanding. This can be accomplished using a diverse array of assessments, from questions targeted at individual students to group evaluations.

Each class starts with a 5 minute “Do Now” question, which serves as a warm up for instruction and review of the previous lesson. The collection of “Do Now” at the conclusion of 5 minutes is a signal for students that the day’s instruction is beginning. Direct instruction is limited to 15 to 20 minutes and despite being introduction of new material, it is student centered. Instead of pure lecturing and simply presenting students with the content, I have the students discover the material and mastery the objectives through guiding questions and activities. I have found this method much more effective for student learning and retention of the material because students obtain a deeper understanding. I also try and supplement the new material with content from previous lessons and other subject areas, in order for students to understand the connections between material and information builds upon each other.

Following direct instruction, the main concepts the students have discovered are reinforced and problems are presented and solution discussed as a class whole in guided instruction. The independent learning varies depending on the content and purpose of the lesson. It either consists of a cooperative learning activity or independent practice. Finally, I use the closing of the class to serve as a reflection and informal assessment. I often have students state in their own words the objectives of the lesson or complete a problem different from the ones presented in class but require the application of the same skills to solve.

Lesson One : *Commutativity: It’s all Around Us* (45 minutes)
Materials Needed: Chalk/white board, chalk/dry erase markers, Promethean Board, chart paper, and markers
Objectives:

1. Demonstrate an understanding of the concept of commutativity
2. Distinguish examples of commutativity in newspapers, journals, articles, and books
3. Create unique examples of commutativity and defend the example with explanation

Standards: Reference appendix

Anticipatory Set/Essential Question: When do we get the same outcome despite a change in order?
Teaching/Presentation: (10 minutes)
Allow students to complete *Do Now* Question (2 minutes)

Do Now Question: If Jose gets 4 books off the shelf and then Anne gets 3, does this have the same result as if Anne gets 3 and then Jose gets 4?

After collection of each student's response to the *Do Now* Question, have students pair up and discuss answers (2 minutes)

Take a class survey of responses and ask one student from each side to provide justification (1 minute)

**The answer to the *Do Now* Question does not have a definite concrete correct answer of yes or no. The answer is subject to how the student interprets the question. For example, if the books were all the same then the answer would be yes, the result would be the same, since there is no difference between the books. However, if the books are not all the same then, in the first situation, Jose will be able to get his favorite book, but there is a possibly Anne would not have that opportunity. Whereas, in the second case, Anne would be able to get her favorite book and Jose might not. Therefore, the answer would be no, the result would not be the same. This question also serves as a foundation for an extension lesson discussing the significance of "the same" and "equals" since in both cases, the end result is a compilation of seven books.

Introduce the concept of commutativity (5 minutes) – project website (http://mathdude.quickanddirtytips.com/commutative-property-of-addition.aspx) using Promethean Board and guide students to understanding the main concept of commutativity.
Guided Practice (8 minutes)
Guide students to formulate an example and non-example of commutativity.

Example: Putting on your shoes. The outcome of having both shoes on does not change despite if the right or left shoe is placed on first.

Non-example: Putting on your shoes and socks. The outcome is different if you place your socks on then our shoes versus your shoes then socks.
Closure (3 minutes)
Have students think and answer the question: Have you ever seen commutativity in mathematics?
Independent Practice (24 minutes)
Create six groups by counting students off 1 through 6 (2 minutes)

Have each group create a T chart on poster paper and create/write three examples and three non-examples of commutativity (7 minutes)

Choose a member from each group to present one example or non-example to entire class (15 minutes)

Lesson Two: *Commutative Property: What does Order have to do With It?* (45 minutes)
Materials Needed: Chalk/white board, Chalk/dry erase markers, decks of cards, and composition paper
Objectives:

1. Describe the Commutative Property in mathematics using proper notation
2. Apply the Commutative Property to solve traditional math problems
3. Evaluate which basic operations (addition, subtraction, multiplication, and division) are commutative

Standards: Reference appendix
Anticipatory Set/Essential Question: When does commutativity hold and when does it fail?
Teaching/Presentation: (21 minutes)
Allow students to complete *Do Now* Question (2 minutes)

Do Now Question: Is the following operation commutative? If so, why? If no, then why not?

To wash your clothes, then dry them

(http://mathforum.org/library/drmath/view/58797.html)

After collection of each student's response to the *Do Now* Question, have a couple of students voluntarily share answers (2 minutes)

Present Commutative Property in mathematics with the aid of Holt textbook, Algebra 1 (2004). Write the following three equations on the white or chalkboard:

1. $2 + 0 = 0 + 2$
2. $1 + 2 = 2 + 1$
3. $3 + 6 = 6 + 5$

Verify with students while the above equations are true (right hand side of the equation equals the left hand side). Present the questions: How is the right side of the equation like the right hand side? How are the sides different? With a partner, have students answer the two questions and create a general formula using variables to demonstrate the Commutative Property. Circulate around the classroom to monitor students' progress. (10 minutes)

Guide discussion as to whether the Commutative Property holds for multiplication (5 minutes)

Present students with formal notation of Commutative Property: (2 minutes)

Addition

For all real numbers a and b: $a + b = b + a$

Multiplication

For all real numbers a and b: $a * b = b * a$

Guided Practice (5 minutes)

Students are presented with equations and are to identify whether it represents the Commutative Property. Examples in the textbook, Holt, Algebra 1, © 2004, can be found on page 87 #32, 36. Teacher should feel free to create his or her own examples.

Closure (2 minutes)

True or False: For all real numbers, a, b, and c: $abc = acb = bca = bac = cab = cba$. Justify your answer.

***The closure question serves as a springboard to an extension of this lesson, if of interest to the teacher. Students can have a discussion about how many different orderings are possible for a given set and how to list all the orders in an organized way to make sure no possibilities are missed. Students can start with four numbers or letters and continue adding one number or letter until a pattern is discovered. This activity foreshadows combinations and factorials.

Independent Practice (17 minutes)

Present the following questions to students:

Do you think that any two numbers can be divided in either order without changing the quotient? Do you think that any two numbers can be subtracted in either order without changing the answer?

A more challenging option: Do I get the same answer if I start with a number, divide it by 3 then by 5 if I start with that same number and divide it by 5 then by 3? Do I get the same answer if I start with a number, subtract it by 3 then by 5 if I start with the same number and subtract it by 5 then by 3? If the answer is yes, explain the extent that the answer yes holds true. Is it for any chosen number?

Have students provide the answer to the questions in groups of 4. You can create the groups using a decks of cards: all the queens are one group, all the kings are another group, etc. Encourage students to substitute with different numbers. Each group of students is to compose a one paragraph (5 sentences) with the answers and supporting evidence to the presented questions. Collect the paragraph at the conclusion of the class for an appropriate grade.

Lesson Three: *Inverse Function: What undoes me?*
Materials Needed: Chalk/white board and chalk/dry erase markers
Objectives:
1. Deeply examine and explore the Commutative Property in the mathematical realm
2. Conclude whether the Commutative Property holds for operations other than multiplication and addition.
3. Create a commutative operation for a pre-existing function

Standards: Reference appendix
Anticipatory Set/Essential Question: If given a function, can I develop a unique a partner commutative operation?
Teaching/Presentation: (14 minutes)
Allow students to complete *Do Now* Question (2 minutes)

Do Now Question: If I have a number, x, is minus 5 from that number then divide by 3 the same as divide that number by 3 and then minus 5? Support your answer.

After collection of each student's response to the *Do Now* Question, have a couple of students voluntarily share answers (2 minutes)

Write the following question on the white or chalkboard: Does there exist other operations that do not hold under the Commutative Property?

Have students answer the following guiding questions to lead to answering the above overarching question:

1. Is 4^3 = 3^4?
2. Is (3/5)x = (5/3)x?

Have students in groups of two, answer the two above questions and the overarching question with their own example. Choose a few pairs to share to rest of class. (10 minutes)
Guided Practice (10 minutes)
Present students with the following process: multiply a number by 2 then add 6. Have the students create a fun name for the process.

Guide students to understanding proper function notation for the process, f(x) = 2x + 6

Present question: Is there a process that commutes with that?

Provide clarifying instructions for students:

1. Start with any number, x (students can begin with choosing a number)
2. Follow the "named" process – multiply the number by 2 then add 6
3. Multiply by some number a and add a number b
4. Next start with the same number, x, and multiply by that number a and add b
5. Then multiply the answer from step 4 by 2 and add 6.
6. What do a and b have to be, for the end of the path to be the same?

Guide students through an example. Let x = 2, a = 5, and b = 3.

1. x = 2
2. 2*2 + 6 = 10
3. 10*5 + 3 = 53
4. 2*5 + 3 = 13
5. 13*2 + 6= 80
6. 53 does not equal 32, therefore a = 5 and b = 3 are not the correct numbers for a commutative process.

A discussion about the definition and significance of inverse functions would be greatly helpful for student understanding during this lesson. Teachers can reference Wikipedia, Mathworld, or other websites. Below I provided a basic explanation of an inverse function and its connection to the Commutative Property.

An inverse function "undoes" the original function. Mathematically, we symbolize the inverse function as f^{-1}. A function, f, takes elements from one set, X, and maps them to elements in another set, Y. The inverse function, f^{-1}, takes those same elements in set Y and maps them to the partnered elements in set X. One can think of an inverse function as a reversible operation because if we choose a number, x, and perform function, f, we will get a result, y and then if we place that number y into the inverse function, f^{-1}, the answer will be the original chosen number, x. We can represent the above as the following:

$$f(x) = y \text{ if and only if } f^{-1}(y) = x.$$

(www.wikipedia.org/wiki/Inverse_function)

Therefore, the lesson asks a student to find the inverse function, f^{-1}, which undoes the original function, f. (f(x) = 2x + 6). If we choose a number, x, multiply it by 2 and add six. What is the inverse function that when we put that answer in, we get back out x?
Closure (5 minutes)
Do you think every operation has a commutative process? Support your answer.

Independent Practice (16 minutes)
Have students guess and check until develop a strategy to develop the answer. Students can work individually or in groups.

One possible answer: a = 1/2 and b = -3

1. x = 2
2. 2*2 + 6 = 10
3. 10*(1/2) -3 = 2
4. 2*(1/2) -3 = -2
5. -2*2 + 6 = 2
6. 2 = 2. Therefore, a = 12 and b = -3 is a correct commutative process

Lesson Four: *Palindromes and Equilibriums: Forwards is Backwards*
Materials Needed: Chalk/white board, chalk/dry erase markers, Promethean Board, composition paper, and colored index cards
Objectives:
1. Examine palindromes and equilibrium chemical equations and the connection to the concept of commutativity
2. Analyze examples of commutativity and non-commutativity in both literature and science
3. Construct own examples of palindromes
4. Solve equilibrium chemical equations

Standards: Reference appendix
Anticipatory Set/Essential Question: Where is evidence of commutativity outside of math class?
Teaching/Presentation: (15 minutes)
Allow students to complete *Do Now* Question (2 minutes)

Do Now Question: What do you notice about the following three numbers? (i.e. do they all have some property in common?)

58285, 484, and 12321

(http://en.wikipedia.org/wiki/Palindromic_number)

After collection of each student's response to the *Do Now* Question, have a couple of students voluntarily share answers (2 minutes)

Explain to students the lesson today will connect the concept of the Commutative Property we learned in the math class to other content areas, particularly English and physical science classes.

Reference the answer to the *Do Now* Question as an introduction to the overarching concept of the lesson: phenomenon that is the same forwards as backwards

Explain to the students we are going to look at two different concepts one in literature and one in science founded on the concept of same forward as backwards. For literature, the term is called palindromes and for science it is equilibrium.

The objective of this lesson is to provide students with an elementary introduction of these two concepts. The goal of the lesson is for students to have a basic understanding of palindromes and equilibrium and its connection to the Commutative Property. Further, students should grasp the cross discipline aspect of the Commutative Property. The objective is not for students to become overwhelmed with the new ideas but rather use them to further enforce and deepen their knowledge of commutativity and strengthen their critical thinking skills.

Present the idea of palindromes in literature and poetry with the following website: http://www.fun-with-words.com/palin_explain.html

Present the idea of equilibrium in science with the following website: http://www.chemguide.co.uk/physical/equilibria/introduction.html

Both websites are excellent resources for providing students with clear and detailed explanations as well as visual supplement. Furthermore, there are different ways you can use the websites in the classroom whether it be for whole class or group instruction. A few examples are below:

1. Project the website on a promethean board and guide students through entire class instruction
2. Place students in groups with computers and do a jig saw with the two websites
3. Place students in groups with computers with guided notes for students to complete as they navigate the websites

Guided Practice (7 minutes)
Choose a few questions from the following website (http://www2.stetson.edu/~efriedma/palindrome/) and guide students to formulate the answers for practice with palindromes.

Present students with a few of the equilibrium problems (teachers can obtain those problems from their respective science textbooks) and guide to answers.
Closure (3 minutes)

The year 2002 is a palindrome. The goal is to determine the five palindromic years prior to 2002 and the next five palindromic ones. Once you have found these years, list them below in ascending order with 2002 in the middle.
(http://www.aimsedu.org/Puzzle/palindromic/palin2.html)

Independent Practice (20 minutes)

Place students in groups of three and provide each group with a palindrome and equilibrium problem. Give each member of the group a role (recorder, monitor, and reporter). The teacher can assign the roles randomly with colored index cards. Definitions of each role are provided below:

1. *Recorder*: writes down all the thoughts, work, and answers of the group
2. *Monitor*: ensures everyone equally participates and finishes the assignment in the provided time frame
3. *Reporter*: shares out the group's work and answers to the class

Additionally, have each group write a one paragraph (minimum five sentences) explaining how commutativity exists in math, science, and English.

Lesson Five: *Problem Solving: Where do I Start?*

Materials Needed: Chalk/white board, chalk/dry erase markers, Promethean Board, and composition paper

Objectives:

1. Identify, compare, and contrast problem solving strategies
2. Solve a variety of problems across the curriculum through application of problem solving strategies
3. Develop critical thinking and communication skills
4. Defend a solution to a presented problem

Standards: Reference appendix

Anticipatory Set/Essential Question: How do I develop a road map through this jungle of a problem?

Teaching/Presentation (12 minutes)

Allow students to complete *Do Now* Question (4 minutes)

> *Do Now* Question: John has exactly $2 dollars in nickels and dimes. She has twice as many dimes as nickels. How many of each does he have?

After collection of each student's response to the *Do Now* Question, have a couple of students voluntarily share answers (3 minutes)

After a few students share their answers, ask a few other students how they started solving this problem. Did they use a particular strategy? If so, was there a specific reason why? (5 minutes)

This lesson presents students to three different problem-solving strategies:

1. Guess and check
2. Try a simpler version of the problem
3. Draw a picture

Students will first be introduced to the strategies with a brief overview. Then, students will apply these strategies to solve a variety of problems. The implementation of the strategies to solve unfamiliar problems across the curriculum and subject disciplines will increase students' critical thinking skills. Further, they will be required to support their solution and chosen problem solving strategy, which will strengthen their communication skills.

Guided Practice (25 minutes)

Write the three problem solving strategies on the chalk/white board:

1. Guess and check
2. Try a simpler version of the problem
3. Draw a picture

Ask students which strategy do they feel would be most efficient to use to solve the *Do Now* Question? The majority of the students will most likely answer the guess and check method.

Provide a brief explanation of each strategy and provide an example of problems when the strategy would be most operative.

1. Guess and check: a strategy when you make an educated **guess** for the answer then **check** to see if it correctly fits the stipulations presented in the original problem.

   Problem:

   Ben knows 100 baseball players by name. Ten are Red Sox. The rest are Blue Jays and Diamondbacks. He knows the names of twice as many Blue Jays as Diamondbacks. How many Blue Jays does he know by name?

(http://www.teachervision.fen.com/math/problemsolving/48896.html?page=1&detoured=1)

2. Try a simpler version of the problem: a strategy when you deconstruct the problem to a more manageable and comprehensible version of the original problem. It is imperative the simpler version of the problem have analogous conditions to the original.

Problem:

How many palindrome numbers are between 0 and 1000?

The teacher can project the website below to guide students through solving the problem using the strategy.

(http://library.thinkquest.org/25459/learning/problem/)

3. Draw a picture: a strategy when you represent the information in the problem by drawing a picture. This strategy is particularly effective for students who are visual learners.

Problem:

Janet and Vicki put up a rope to mark the starting line for the sack race. The rope was 10 meters (m) long. They put a post at each end of the rope and at every 2 m. How many posts did they use?

(http://library.thinkquest.org/25459/learning/problem/)
Closure (3 minutes)
Which of the three strategies do you find easiest to use? Hardest? Explain your answers.
Independent Practice (15 minutes)
Have students form groups of three. Have each group choose a problem from the previous lesson (entitled: *Palindromes and Equilibriums: Forwards is Backwards*). Then have each group write a paragraph (minimum 5 sentences) including the following:

1. Choose one of the three strategies that you feel would be the best to solve your selected problem
2. Explain why you think that is the best strategy
3. Explain how you would implement that strategy to arrive at the answer

After (about 10 minutes), pair each group with one other group. Have the groups take turn sharing their paragraphs.

Lesson Six-Twelve: *Mini Lesson: Your Turn to be the Teacher*
Materials Needed: Chalk/white board, Chalk/dry erase markers, composition books (journals) – one for each student, computers, composition paper, and index cards
Objectives:

1. Understand the expectations of the final assessment/presentation
2. Apply research strategies to effectively collect information
3. Assemble a journal to track progress throughout the project
4. Prepare and organize a presentation defending a solution to a series of commutativity inter-disciplinary problems
5. Evaluate other students work using a student and teacher devised rubric

Standards: Reference appendix
Anticipatory Set/Essential Question: How can I formulate a convincing argument to support my solution?
Teaching/Presentation (12 minutes)
Allow students to complete *Do Now* Question (3 minutes)

Do Now Question: Summarize the concept of commutativity in your own words.

After collection of each student's response to the *Do Now* Question, have a couple of students voluntarily share answers (3 minutes)

Distribute the rubric for the final assessment. An example of the rubric to use can be found on the following website (http://ed.fnal.gov/lincon/w01/projects/library/rubrics/presrubric.htm). I like this rubric in particular because of its diversity and number of components. However, feel free to add or remove components or construct a completely novel rubric.
Rubrics are important to provide to students, particularly for project-based assessments, in order for students to gain a clear understanding of the expectations of the assessment. Furthermore, the rubric enables for straightforward grading. It is clear to students and the teacher the grade assigned to each student. Allow students a few minutes to preview the rubric and ask any questions.

Write the agenda for the lesson on the chalk/white board as well as provide students with a checklist for the presentation. I provided a draft of a checklist below as a reference. This helps guide and focus students. Also, explain to students they will have a number of periods to complete the assignment and a couple of classes will be devoted to presentations of the final work. (Lessons numbers 7 through 12)
Student Checklist for Final Project

1. Appropriately research presented problems
2. Properly cite a minimum of five resources
3. Compose a journal of entries documenting advancement through the assignment
4. Formulate solutions to the problems
5. Write a oral presentation which presents the problems and defends the solutions
6. Practice the presentation
7. Create a visual aid (poster, PowerPoint slide show, etc)
8. Share the presentation with classmates
9. Evaluate your own project as well as other classmate's work

Guided Practice (10 minutes)
Provide each pair of students with a computer and a number of problems dealing with commutativity inter-disciplinary problems. The teacher can incorporate problems dealing with the Commutative Property, palindromes, and/or equilibrium chemical equations. However, I believe the project will most benefit students if presented with problems

centered on commutativity concepts unfamiliar to them. Guide students through proper research techniques such as search engines, etc

Closure (3 minutes)

Write down one useful resource you found today and one possible way you will incorporate it into your final product/assignment.

Independent Practice (20 minutes)

Allow students time on their computers to navigate the Internet and research their problem using various research engines. Emphasize to students the importance of properly citing their resources. An excellent resource to direct students to for proper citing is: (http://library.duke.edu/research/citing/workscited/). In addition, students can write a few paragraphs in their journals to document their findings and keep track of their progress in their final assignment.

***In lessons 7-12, groups should continue to research, formulate solutions, and write in their journals. Also, students should write and practice their oral presentation of introducing the problems and defending the solutions. Teachers should strongly encourage students to also create a visual aid for the presentation (poster, PowerPoint slide show, etc). Finally, the last two lessons (11 and 12) should be devoted to the presentations and student assessments.

Annotated Bibliography/Resources

Teacher Bibliography

http://www.math.upenn.edu/~deturck/probsolv/main.html
The website was created by Professor Dennis DeTurck, Professor of Mathematics and Dean of the College of Arts and Sciences at University of Pennsylvania. The website is a directory of additional math problems directly connected to a specific problem solving strategy (a few incorporated into this curriculum). These problems can be given to students on “down-time” during the year or as an extension of a math lesson.

http://www.yale.edu/ynhti/curriculum/units/2004/5/04.05.06.x.html
The website is a curriculum by teacher who attended Yale-New Haven Teachers Institute in 2004. I particularly chose this curriculum as a resource because it links to my curriculum with implementing problem solving strategies to solve math problems. Additionally, I really like how the curriculum includes a database of problem written by students. This can serve as a catalyst for an extension lesson for students. After instructing the students on problem solving strategies, the teacher can present these student created problems and ask them to apply the strategies, solve the problems, and then create their own.

http://www.humboldt.edu/~tha1/hunter-eei.html

The website is an excellent outline of the well known Madeline Hunter Model of the seven step lesson plan used by many teachers as the format for instruction. . This website is a helpful resource because it provides a cohesive and clear explanation of the elements to each step. It also includes a summary at the bottom of the webpage. I used this particular seven-step format to create the lesson plans in the curriculum.

http://mathchaostheory.suite101.com/article.cfm/commutativity_in_mathematics_and_nature
The website provides a number of examples of commutativity in mathematics as well as life and the natural world. Teachers can reference this website if they are having difficulty formulating examples of commutativity and non-commutativity in real life.

http://ed.fnal.gov/lincon/w01/projects/library/rubrics/presrubric.htm
The website offers a number of rubrics for teachers to utilize when assessing students' work. There are general rubrics that serve as an outline as well as rubrics for specific assignments such as oral presentation. Teachers should feel free to modify the website rubrics as needed to better fit the grade level, subject, and assignment. I chose the one specific to oral presentations to use as a base for students to assess their final project.

Though I did not directly incorporate these works into my curriculum, I believe they are extremely helpful resources for all teachers. Therefore, I wanted to include them as references to explore as needed or desired.

1. Allen, Janet. "Inside Words: Tools for Teaching Academic Vocabulary: Grades 4-12." Portland, ME: Stenhouse Publishers, 2007.
2. Blythe, Tina and Associates. "The Teaching for Understanding Guide." San Francisco, CA: Jossey-Bass, 1998.
3. Tate, James, Schoonbeck, John. "Reviewing Mathematics." New York, NY: Amsco School Publications, Inc., 2003.
4. Tomlinson, Carol Ann. "The Differentiated Classroom: Responding to the Needs of All Learners." Alexandria, VA: Association for Supervision and Curriculum Development, 1999.
5. Tomlinson, Carol Ann. "Fulfilling the Promise of the Differentiated Classroom: Strategies and Tools for Responsive Teaching." Alexandria, VA: Association for Supervision and Curriculum Development, 2003.

Student Bibliography

http://mathworld.wolfram.com/
The website is a well-known resource that can serve well as a glossary for students. Students can easily search a mathematical concept of word and the website provides a comprehensible definition as well as examples and non-examples.

http://www.purplemath.com
The website is a resource for both teachers and students. For teachers, the website contains pre-made lessons on algebraic concepts that are "student-friendly". I typically reference this website when I am teaching a concept in Algebra 1 or 2 and need an additional means of presenting the material. For students, the website provides homework help as well as a study skills survey for students to access their study habits.

http://www.fun-with-words.com/palindromes.html
The website is an excellent introduction for students on palindromes. I suggest referencing and guiding students through the website as a means to convey the idea of palindromes. The website provides a thorough definition, examples, and extensions on the basic palindrome.

http://mathdude.quickanddirtytips.com
The website is a phenomenal database for mathematics with visual as well as auditory explanations. In particular, the website is extremely student-friendly, with breaking down complex concepts and providing relatable examples. I used this website to present the concept of Commutative Property of Addition (http://mathdude.quickanddirtytips.com/commutative-property-of-addition.aspx);

http://mathforum.org/library/drmath/view/58797.html
The website is part of the Math Forum, Ask Dr. Math, hosted by Drexel University. The website is a post by a student with a question about Commutative Property examples and non-examples. The examples and non-examples can be implemented in the curriculum for students to answer. The general website, http://mathforum.org/dr.math/, is an excellent resource for students because it contains questions posted by students and answered by math majors and professors.

http://en.wikipedia.org/wiki/Palindromic_number
The website provides a general definition of palindrome as well as examples in math and beyond.

http://www.chemguide.co.uk/physical/equilibria/introduction.html
The website offers an excellent introduction for students to the concepts of chemical equations and equilibrium. The explanation includes examples and visual representations. I suggest guiding students through the website in order for them to gain a solid understanding of equilibrium and its connection to the Commutative Property.

http://www2.stetson.edu/~efriedma/palindrome/
The website is a puzzle worksheet consisting of a list of riddles where the answers are all palindromes. This is a fun activity for students, which involves palindromes.

http://www.aimsedu.org/Puzzle/palindromic/palin2.html
The website presents a problem that incorporates commutativity as well as palindromes by asking for years that are palindromes. I used this problem as my closure question in Lesson Four.

http://www.teachervision.fen.com/math/problemsolving/48896.html?page=1&detoured=1
The website is an excellent explanation of the problem solving strategy guess and check. Furthermore, the website includes problems which can be most effectively solved using the guess and check method. I incorporated a few of these problems in Lesson Five.

http://library.thinkquest.org/25459/learning/problem/
The website contains several problem solving strategies as well as problems solved by implementing those strategies. I utilized a number of those problems for students to answer in Lesson Five.

http://library.duke.edu/research/citing/workscited/
The website is a resource for students to determine how to properly cite resources in research papers or projects. I particularly like this website because it contains a variety of formats from MLA to APA to Chicago and others. Furthermore, it includes a vast number of choices for sources such as books, journals, newspapers, and websites.

Appendix

State Standards - Pennsylvania

Mathematics
2.1.8.B: Simplify numerical expressions involving exponents, scientific notation and using order of operations.
2.2.8.B: Add, subtract, multiply and divide different kinds and forms of rational numbers including integers, decimal fractions, percents and proper and improper fractions.
2,4.8.A: Make conjectures based on logical reasoning and test conjectures by using counter-examples.
2.4.8.B: Combine numeric relationships to arrive at a conclusion.
2.5.8.A: Invent, select, use and justify the appropriate methods, materials and strategies to solve problems.
2.5.8.B: Verify and interpret results using precise mathematical language, notation and

representations, including numerical tables and equations, simple algebraic equations and formulas, charts, graphs and diagrams.
2.5.8.C: Justify strategies and defend approaches used and conclusions reached.
2.5.8.D: Determine pertinent information in problem situations and whether any further information is needed for solution
2.8.8.B: Discover, describe and generalize patterns, including linear, exponential and simple quadratic relationships

English
1.3.8.C: Analyze the effect of various literary devices such as sound techniques (e.g., rhyme, rhythm, meter, alliteration) and figurative language (e.g., personification, simile, metaphor, hyperbole, allusion).

Science
3.1.10.E: Describe patterns of charge in nature, physical, and man made systems: Recognize that stable systems often involve underlying dynamic changes (e.g. a chemical reaction at equilibrium has molecules reforming continuously).