The Easy Way to Calculate: The Art and Craft of Problem Solving

Krishan L. Wadhwa

University City High School

Overview:

In this unit, students will explore strategies to solve mathematical problems from different aspects of real-world life. Solving problems in any branch of mathematics is an art. From the Stone Age through Einstein's age, mathematical ideas have played a dominant role to simplify calculations. Mathematics, being the spinal chord of physical sciences, develops different strategies to solve problems. For example, at one time architects and engineers required hours and hours to do calculations using slide rules and logarithm tables, but now the same calculations are done just at the click of a few keys on computers and calculators. Indeed, modern software often performs the calculations automatically, without any guidance from the user whatsoever.

In solving mathematical problems, skills and strategies are always necessary. A student's success in solving a problem depends on the student's, having, acquired relevant information prior to encountering the problem and on the student's critical thinking skills. A student's talent lies in understanding the problem and planning the approach, completing of work and interpreting of the result.

The most basic mathematical tool is counting. The art of recording numbers, or numeration, was invented to meet the needs of society as it became more complex than that of ancient herdsmen. The numbers now in use, (0,1,2,3,4,5,6,7,8,9) have been in common use for only about a thousand years. They are known as Hindu-Arabic figures, because they were originated in India and were introduced to Europe by the Arabs.

Since the earliest times particular numbers have had a strange fascination. Certain numbers have been believed to be endowed with mysterious qualities, and even nowadays many believe that everyone has "lucky" and "unlucky" numbers. Astrologers and astronomers connect these numbers with the zodiacs of people. Large group of societies are astonishingly convinced with the superstition of number “13”. This superstition is so generally rooted that in many office buildings the thirteenth floor is either missing or numbered as 12A and Friday the thirteenth is almost National Hoodoo Day. In the eyes of a mathematician, there is nothing supernatural in numbers. Rather, each number has its own arithmetic properties. Besides the number 10, mathematician’s favorite number, 12 was most commonly used as a basis for counting in ancient times. Although now we use 10 as our basis for enumeration, there are many remnants of base-12 counting in modern life: There are twelve months in a year, 12 inches in a foot, twelve items in a dozen, twelve dozens in a gross. Nowadays, we use the decimal system in our calculation, (probably because we have ten fingers on our hands, and the first step in counting were performed on our fingers, literally “digitally”). But base-10 is not, in some ways, as convenient as the base-twelve system. For example 10 is divisible by 1,2,5 and 10 only, while 12 is divisible by 1,2,3,4,6,and 12, so it is often more convenient to divide sets of 12 equally among groups of people. So, there are fewer unit fractions of 10 (namely 1/2 and 1/5) than unit fractions of 12 (1/2, 1/3, 1/4, 1/6).

Nowadays, there is a great deal of familiar talk about big numbers such as million and billions and trillions (think about state or federal budgets or the amount of oil being spilled in the Gulf of Mexico). But, especially in science, we have needed to express numbers much larger than these. When we talk about the number of hairs on our head or number of grains of sand in one kilogram of sand, we are at a loss to count them. Scientists have devised a method to count these large numbers by using scientific notation, in which one expresses a large number by reporting only the first few digits of the number and how many digits are required to write the number. For example, Avogadro’s number (the number of atoms or molecules in a “mole” or molecular weight of a substance) is equal to 6.022* 10^23 particles. This means that 6022 are the first few digits in this number, and it would require 24 digits to write it out. To get some idea of how large Avogadro’s number is, imagine that every living person on Earth (about 6 billion people) started counting the number of atoms of 1 mole of carbon. If each person counted nonstop at a rate of one atom per second, it would take 3 million years to count every atom.

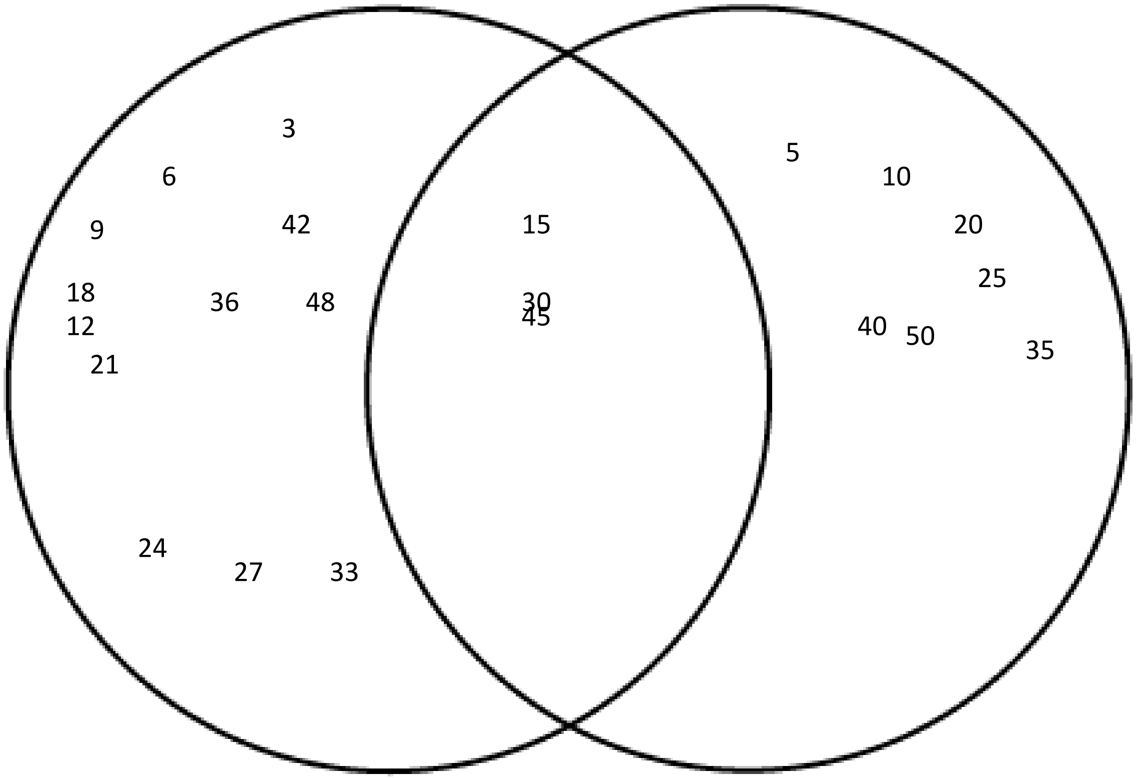
Beyond enumeration, we have needed to combine numbers using the familiar arithmetic operations. Some of the methods of rapid calculation are so fascinating that many people considered them to be almost magic. However with the application of Algebra these mathematical calculations can be understood, and by applying Algebra, solving problems in many branch of mathematics or science has became an art.

Not only the large numbers, but fractions also play a dominant role in mathematics and science. Fractions were known to Egyptians around 3000 BC and they had an interesting way to represent fractions, different from the numerator/denominator method we use today. They always started with fractions that can be represented as reciprocals of natural numbers, or unit fractions, such as 1/2, 1/3, 1/4, 1/5 etc and they were expressed other fractions as sums of unit fractions where all the unit fractions were different. For example: 2/3= 1/2+1/6; 4/5 = 1/2+1/5+1/10. It is a remarkable fact that any (positive) rational number can be expressed in this way.

This unit will enable the students to use different strategies such as Venn-diagrams, graphs, tables, and conceptual maps used at 9-12 levels. Emphasis is also given to the problems solving skills through mathematical modeling. Using these skills students taking standardized tests like SAT's Praxis 1 & 2, GMAT etc. can save lot of time and use it to review the tests.

One of the strategies in solving counting problems involving two or three sets of related objects is the Venn diagram. A Venn diagram is a pictorial representation of sets in which each set is represented by an enclosed region. The regions common to two or more of the enclosed regions in a Venn diagram shows the intersection of sets, or elements that belong to both or all of the sets. For example, the number of positive integers between 1 and 50 that are divisible by 3 or 5 or both can be solved using a Venn diagram. Let A be the set of numbers between 1 and 50 which are divisible by 3 and B be the set of numbers divisible by 5. If we picture A and B as a pair of intersecting circular regions, the common region represents the set of numbers divisible by both 3 and 5. This Venn diagram not only represents the set of numbers that are divisible by 3 or 5 or both but also the numbers that are divisible only by 3 or only by 5.

VENN DIAGRAM



From the Venn diagram, we can list:

C = the set of numbers divisible by 3 only, and not by 5) =

{ 3,6,9,12,18,21,24,27,33,36,39,42,48}. There are 13 numbers that are divisible by 3 only. In other words the cardinal number of set C is 13. The usual notation for the cardinality of a set (i.e., the number of elements in it) is n(C), and so we write n(C)= 13.

Likewise, the set D = {numbers between 1 and 50 divisible by 5 but not by 3} =

{ 5,10,20,25,35,40,50}. There are 7 numbers in this set, and so we write n(D) =7.

There are 3 numbers between 1 and 50 divisible by both 3 and 5. This set is represented by $A \cap B$ (the intersection of sets A and B), and so $n(A \cap B) = 3$:

$$A \cap B = \{15, 30, 45\} \quad \text{and so} \quad n(A \cap B) = 3.$$

Thus, the total number of positive integers between 1-50 (inclusive) divisible by 3 or 5 or both is:

$$13 + 7 + 3 = 23.$$

Venn diagrams have wide applications in real-world situations and help to make enumeration calculations simpler. There are several calculation tools one uses for this purpose, such as

(i) Probability of Compound Events: P(AUB) =P(A) + P(B) – P(A∩B). Here perhaps you mean the "Principle of Inclusion and Exclusion", which states that

n(AUB) = n(A) + n(B) – n(A∩B) – since we're not talking about probability here, we should stick to cardinality. And, since you use it later, you should mention its generalization to three (or more) sets

(i) Arguments and Venn Diagrams: Example: determine the validity of the following argument.

   a. S1: All my friends are educators.
   b. S2: dickens is my friend.
   c. S3: None of my neighbors are educators.
   d. S4: Dickens is not my neighbor

   Using Venn Diagrams, it can be proved that S is a valid conclusion and so the argument is valid.

(ii) Rules for enumerating the result of set operations such as: union and intersections of sets; complement of a set etc. Many verbal statements can be described by Venn diagrams; hence, Venn Diagrams can sometimes be used to determine whether or not an argument is valid.

<u>Venn Diagrams are useful in a number of contexts, for example</u>:

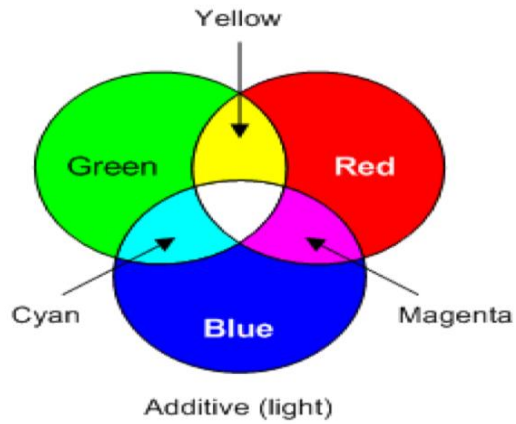
(i) To get a correct count in problems involving multiple languages, patients in a hospital administered multiple medicines, people reading multiple magazines and newspapers in a community etc.

(ii) During World War I, a system of classifying military patients was designed to help allocate medical supplies. This system was given the name TRIAGE, a French word meaning to chose or sort. Physicians divided patients into three categories: (a) those too severely wounded to survive, (b) those who would survive only with immediate medical attention, (c) those who received the majority of resources available. Venn diagrams can be used to assist physicians with this category of medical problems.

(iii) Venn diagrams have wide applications in mixing of colors. The combination of the additive primary colors in any two circles produces complementary color of the third additive primary color. Likewise the combination of the subtractive primary colors by any two filters produces the complementary color of the third subtractive primary color. Venn diagram illustrations of mixing of these colors provide a powerful visual aid. The names of the primary additive and subtractive colors can be represented by following equations:

Red + Green = Yellow,   Green + Blue = Cyan,   Blue + Red = Magenta

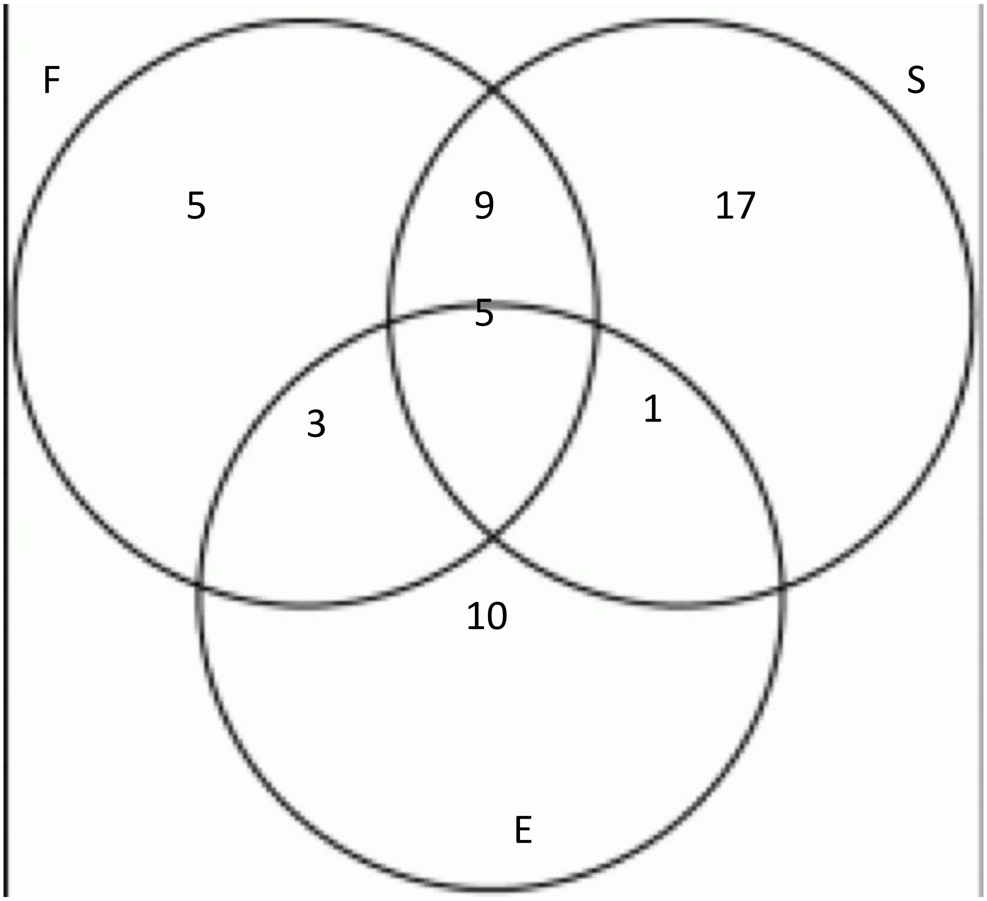
Here red is complementary to cyan, green is complementary to magenta and blue is complementary to yellow.



Example: In a multilingual group of students, 5 students speak all three of the languages French, Spanish and English; 14 of them speak French and Spanish; 6 of them speak Spanish and English; 8 of them speak English and French; 10 of them speak only English; 5 of them speak only French; 17 of them speak only Spanish. How many people are there in the group?

Solution: After assigning the different sets to represent each of three languages and cardinality of these sets, a Venn diagram can be drawn to solve the problem easily. In this situation n(F), n(E) and n(S) are not given, however we know that $n(F\cap E)=8$, $n(E\cap S)=6$, $n(S\cap F)=14$, and $n(E\cap S\cap F)=5$. This problem can be solved easily by the use of a Venn diagram. Let F, S and E be the three circles representing French, Spanish and English speaking respectively, drawn as shown below. Since 5 of them speak all the three languages, this set of students is represented can be shown by the region common to all three circles. The rest of the numbers can be filled accordingly and after adding these numbers we get the required number of students in the group i.e., 50.

VENN DIAGRAM



Another strategy of solving this problem is by using the principle of inclusion and exclusion from discrete mathematics, using the formula:

$$n(F \cup S \cup E) = n(F) + n(S) + n(E) - n(F \cap S) - n(S \cap E) - n(E \cap F) + n(F \cap S \cap E)$$

$$= 22 + 32 + 19 - 14 - 6 - 8 + 5 = 50.$$

Note: Here n(F) means number of students who can speak French, even if they can speak other languages as well. It doesn’t mean the number of students who speak only French.

Exercise 1**:** In a survey of 70 people in India, it was found that 25 people read *Indian Express*, 32 people read *Times of India*, and 30 read *Hindustan Times*. Moreover, 10 read both *Indian Express* and *Hindustan Times*, 12 read both *Indian Express* and *Times of India*. 9 read *Times of In*dia and *Hindustan Times*, and 5 read all the three newspapers:

(a) Find the number of people who read at least one of the three newspapers.

(b) Find the number of people who read exactly one newspaper.

(c) Find the number of people who do not read any newspaper at all.

Answer:

a. 61

b. 40

c. 9

***Exercise 2*:** In a survey of 100 students at University of Pennsylvania, it was found that 32 students study Math, 20 students study Physics and 45 students study Biology. On further analysis it was found that 15 students study both Math and Biology; 7 students study both Math and Physics and10 students study both Physics and Biology. But 30 of these 100 students do not study any of the three subjects at all. Determine

(a) The number of students studying all the three subjects

(b) The number of students studying exactly one of the three subjects.

Answer:

a. 5

b. 48

EXERCISE 3: The staff in a clinic of 120 patients has diagnosed two diseases: 75 of the patients have disease A which requires an antidote and no other care for recovery; 55 of the patients have disease B which require intensive therapy. Some of the 120 have both diseases and constitute an overlap group that can be comforted, but not cured. How many patients have both diseases? How many patients should receive intensive therapy?

Answer:

a.10

b.45

Strategies:

1. Prime numbers.
2. Venn diagrams.
3. Egyptian way of writing fractions.

After understanding the problem, students must choose an appropriate strategy to solve the problem. The strategy one chooses is the plan of action for solving the problem. Several strategies are developed in this unit for different topics to help students plan their approach.

Prime Numbers

Problems involving prime numbers have fascinated mathematicians from since the time of the ancient Greeks. Many attempts have been made to write a formula that produces all the prime numbers, but after working on number of steps, the formula fails or it skips some prime numbers. For example, a series of prime numbers can be generated by using formula. $P= a^2 \pm a*b + b^2$, where a and b is two consecutive positive integers:

For a=1, b=2; $p=1^2+1*2+2^2=7$

$1^2-1*2+2^2=3$

For a=2, b=3; $2^2+2*3+3^2=13$

$2^2-2*3+3^2=7$

For a=3, b=4; $3^2+3*4+4^2=37$

$3^2-3*4+4^2=13$

For a=4, b=5; $4^2+4*5+5^2=61$

4^2-4*5+5^2=41

However, for a=5, b=6; 5^2+5*6+6^2=91 (not prime)

5^2-5*6+6^2=31

For a=6, b=7; 6^2+6*7+7^2=127

6^2-6*7+7^2=43

Another miss comes for a=7, b=8; 7^2+7*8+8^2=169 (not prime)

7^2-7*8+8^2=57 (not prime)

And for a=8, b=9; 8^2+8*9+9^2=217 (not prime)

8^2-8*9+9^2=73

As the number become bigger and bigger, primarily is lost and numbers produced become composite more and more often. It is conjectured that there is no known formula that yields all of the primes or composite.

The motivation for trying to use this formula to generate primes comes from algebra, when we factor expressions such as (a^3-b^3) or (a^3+b^3):

(a^3-b^3)= (a-b)(a^2+ab+b^2)

(a^3+b^3)= (a+b)(a^2-ab+b^2)

As noted above, assigning different, consecutive integer values to a and b, it seems that $a^2 \pm ab+b^2$ gives rise to a sequence of prime numbers but after few attempts it fails. Despite trying several methods of generating such formulas, many fundamental questions about prime numbers remain open. For Example:

(a) Even number bigger than two is the sum of two primes. (8=3+5, 16 = 5+11; 10=3+7). This is called Gold Bach's Conjecture .
(b) There are infinitely many twin primes (pair of primes whose difference is two). 5-3=2, 7-5=2, 13-11=2, 19-17=2, 73-71= 2.

Despite the apparent mysteriousness of the prime numbers, there are interesting problems involving primes that are accessible and interesting to students. At the root of most of them are basic facts about primes, such as their definition (a prime is divisible by only itself and by 1) or the fact that if a product of two numbers, say a times b, is divisible by a prime number p, then it must be true that either p divides a or p divides b (or both). In abstract algebra (ring theory), this is sometimes taken as the definition of a prime element of a ring.

Some interesting problems involving prime numbers:

***Example*:**

Suppose x and y are two positive integers defined by $x= \sqrt{(z+43)}$, $y=\sqrt{(z-10)}$ (it is not necessary to assume that z is an integer). What are the possible values of x and y?

Solution: $x= \sqrt{(z+43)}$

$x^2= z+43$------------(a)

$y= \sqrt{(z-10)}$

$y^2= z-10$--------------(b)

Subtracting equation (b) from equation (a) gives

$x^2-y^2=53$

$(x-y)(x+y) = 53$

Since x and y are integers that multiply together to give 53 (a prime number), the only possibilities for x and y are 1 and 53. And since we assume x and y are positive, then we must have

$x- y = 1$

$x+ y = 53$

$x= 27, y= 26$

Exercise 1: Suppose x and y are two positive integers defined by x= $\sqrt{(z+27)}$ and y= $\sqrt{(z-10)}$. What are the possible values of x and y?

Exercise 2: Suppose x and y are two positive integers defined by x= $\sqrt{(z+77)}$ and y= $\sqrt{(z-12)}$; what are x and y?

Note: Problems of this kind can be designed by expressing a prime number as the sum of two positive integers (i.e., 27+26=53, 27+10=37, 77+12=89). Once one sees this pattern, a more advanced question would be whether it always works – but as is often the case with statements involving prime numbers, the prime 2 is an exception, for example: 2 is prime; 1+1=2. But if we go through the calculation for x and y we find

$x= \sqrt{(z+1)}$, $y= \sqrt{(z-1)}$, $x^2=z+1$, $y^2=z-1$,

$x^2-y^2= 2$; $(x-y)(x+y) =1*2$; $(x-y)=1$; $(x+y)=2$

Solving for x & y, we get x=3/2 & y= 1/2.

Because x and y are not integers, so the general pattern does not hold in this case (number theorists sometimes get around this kind of exception by saying that some results hold true only for "odd primes").

Problems based on prime numbers can sometimes be understood by using divisibility rules. To check whether a number of modest sizes are prime or not, students can use a strategy such as the following:

(i) Does the number end with 0, 2, and 4,6,8,5?

The number cannot be prime (unless it happens to be equal to 2). Any even numbers other than 2 are composite (being divisible by 2) and a number ending in 0 or 5 is divisible by 5 (and hence not prime unless it happens to be equal to 5). So primality is lost.

(ii) Does the number end with 1, 3,5,7,9? Then check for the divisibility by 3. Add all the digits of the number and check whether the sum is divisible by 3 (this can be repeated recursively for very large numbers). For example 123,456,789 is a big number:

Sum= 1+2+3+4+5+6+7+8+9=45

New Sum= 4+5=9 is divisible by 3. Therefore 123,456,789 is divisible by 3 and hence is not prime.

The tests given so far check for divisibility by 2, 3 and 5. They will detect many composite numbers, but will fail to show that a product of two larger primes, such as 11 times 13, is composite. It turns out that the ability to find large prime numbers, and to factor very large numbers (hundreds of digits) into primes is important in computer security [add reference on RSA cryptosystem?].

The only truly foolproof way to check whether a number is prime is to attempt to divide it by all prime numbers less than or equal to its square root (which requires one to have a list of all the prime numbers). For example, to determine whether the number 3413 is prime, we must divide 3413 by the prime numbers less than 58, i.e., by 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, and 53. Because 3413 is not exactly divisible by any of these prime numbers, it is a prime number.

***Egyptian Way of Writing a Fraction*:**

Sometimes it is quite fascinating to work on fractions. Technology has made it easy to deal with addition, subtraction, multiplication and division of fractions. But if we investigate the history of the methods for writing and combining fractions, we soon encounter the methods used by the ancient Egyptians. Basically, the Egyptians' way of writing fractions is to express a fractional quantity only by adding distinct unit fractions.

It can be an interesting project for students to try and discover a reliable and efficient method for expressing an arbitrary fraction in the Egyptian system. Doing so will provide practice in adding fractions and in organizing their thinking about a challenging, somewhat open-ended problem.

In the end, to write any fraction in Egyptian way, the easiest method is to write an equivalent fraction by multiplying the numerator and denominator of a fraction by the same non zero number and splitting the numerator in distinct factors of denominator.

To explain the method to my students, who have no idea of recursion formulas, I start by dividing the proper fractions into two categories:

(1) Proper Fractions more than one-half, for example 2/3,4/5,11/17,5/8. Divide these further into two categories: those with an odd integer in the denominator; and those with an even integer in the denominator. If there is an odd integer is in the denominator:

   (a) Multiply numerator and denominator by 2 or 4 or 8…….
   (b) Partition the new numerator into factors of new denominator (this means "express the numerator as a sum of distinct factors of the denominator [how do we know that this is always possible?]).
   (c) Reduce the addends obtained in this way to the lowest terms.

Example: Write in Egyptian way (a) 11/17 (b) 5/8

Solution:

   (a) 11/17= 11*8/ 17*8= 88/136 =(68+17+2+1)/136 =68/136+17/136+2/136+1/136=1/2+1/8+1/68+1/136
   (b) 5/8 =(4+1)/8 = 4/8+1/8 =1/2+ 1/8

(2) Proper fractions less than one-half, such as: 5/11, 3/7, 5/14/7/33…………

Follow the following steps to write fractions in Egyptian way:

   (a) Split up numerator as the sum of the factors of the denominator; else multiply numerator and denominator by 2 or 3 or 4…………
   (b) Split up new numerator as the sum of factors of new denominator (reword – again, how do we know that this is always possible?).
   (c) Reduce the fractions to the lowest terms (i.e. unit fractions).

Examples:

7/33=(7*3)/(33*3)=21/99=(11+9+1)/99=11/99+9/99+1/99=1/9+1/11+1/99.

Note: The representation of a fraction in the Egyptian way is sometimes not unique. For example, 11/15 can be represented in two different ways:

11/15=11*2/15*2=22/30=15/30+5/30+2/30=1/2+1/6+1/15 or

11/15=11*3/15*3=33/45=15/45+9/45+5/45+3/45+1/45=1/3+1/5+1/9+1/15+1/45.

APPLICATIONS OF EGYPTIAN WAY OF WRITING FRACTIONS:

(1) To divide odds among evens:- Suppose we have 5 pies of pizzas to share among 8 people. How can we divide it evenly, without using a calculator? Of course, the quickest answer is 5/8th of a pie to each. But how it is done practically? The Egyptian way of representing a fraction provides a solution. For example: 5/8=4/8 + 1/8 = 1/2 + 1/8, so each of the eight people gets one-half of a pizza pie (four pizza pies are divided into halves) and one eight of the fifth pizza is given to each person.

(2) To compare fractions (i.e., to tell which of two fractions represents a larger number) - Various ways can be used to compare fractions. For example, we could just convert the fractions into decimals and decide which fraction is bigger or smaller (especially if we have access to a calculator). For example, to decide which is bigger 2/3 or 3/4., we convert 2/3 =0.66666………. and 3/4 = 0.75. We can decide that 3/4 is bigger than 2/3.Another way of comparing of fractions is to write equivalent fractions with the same denominator, 2/3 = 8/12 and 3/4 = 9/12. We can decide that ¾ is bigger than 2/3.

The Egyptian way to compare fractions is to convert the fractions into sums of unit fractions. For example, 2/3 = 1/2+ 1/6 and 3/4 = 1/2 + 1/4. We conclude that ¾ is bigger than 2/3. Another example: Which is greater, 3/4 or 4/5?

3/4= 15/20 =10/20 + 5/20 = 1/2 + 1/4

4/5 = 16/20 = 10/20 + 5/20 + 1/20 = 1/2 + 1/4 + 1/20

We can thus conclude 4/5 is greater than 3/4 exactly by 1/20.

Lesson Plan 1:

This lesson plan is designed for my discrete math class. This class is a homogenous mixture of 12^{th} graders (i.e. seniors).

Topic:

Prime Numbers and Composite Numbers.

State Mathematics Education Standards Covered:

- 2.1- Numbers, Number systems and Number Relationships. Types of Numbers (e.g.; whole, prime, irrational, complex). Equitant Forms (e.g.; fractions, decimals, percents.
- 2.1.5: E: Explain the concepts of prime and composite.
  G: Develop and apply number theory concepts (e.g.; primes, factors, multiples, composites) to represent numbers in various ways.
- 2.1.11: Use operators (e.g.; opposite, reciprocal, absolute value, raising to a power, finding roots, finding logarithms.)
- 2.2.12: Computation and Estimation. Basic functions (+, - . x, /)/ reasonableness of answers/calculators.

Objectives:

(a) Students can understand and apply the definitions of prime numbers and composite numbers.

(b) Students are able to list prime numbers between 1-100 and determine the primness of big numbers.

Pre-requisite Knowledge:

Students are already familiar with the divisibility of numbers such as 2,3,4,5,6,7,8,9,10. Students already know that first prime number is 2 and it is the only even prime number. Composite numbers start at 4 and all even numbers are composite because they are all divisible by 2.

Time Schedule:

Complete topic will be covered in 2 periods of 50 minutes each.

Procedure:

(i) Students are asked to write all the prime numbers from 1-20.

(ii) After checking that they have done this successfully, the teacher asks them to extend their lists another 20, i.e. 1-40, 1-60, 1-80, 1-100.

(iii) Numbers are subject to testing using divisibility criteria.

(iv) Students are asked to review and understand the factorization of expressions such as:

$(a^3 - b^3) = (a - b)(a^2 + ab + b^2)$

$(a^3 + b^3) = (a + b)(a^2 - ab + b^2)$

(v) Students are asked to assign different positive integers in the second factor of above expressions and check the result for primality.

(vi) Students are asked to conjecture formulas for prime numbers in algebraic domain and test their validity.

(vii) Students are given some interesting exercises on prime numbers.

Lesson Plan 2

Topic: Applications of Venn diagrams.

Standards: State Mathematics Education Standards:

- 2.1- Numbers, Number systems and Number Relationships. Types of Numbers (e.g.; whole, prime, irrational, complex). Equitant Forms (e.g.; fractions, decimals, percents.
- 2.1.5: E: Explain the concepts of prime and composite.
  G: Develop and apply number theory concepts (e.g.; primes, factors, multiples, composites) to represent numbers in various ways.

- 2.1.11: Use operators (e.g.; opposite, reciprocal, absolute value, raising to a power, finding roots, finding logarithms.)
- 2.2.12: Computation and Estimation. Basic functions (+, - . x, /)/ reasonableness of answers/calculators.

Objectives:

(i) Students are to understand the method of drawing Venn diagram.

(ii) Students are to follow correct procedures for entering data in circles drawn in a Venn diagram.

(iii) Students should understand when the use of Venn diagrams is an appropriate problem-solving strategy.

*Pre-requisites Required***:**

Students are familiar with Set Theory and Operations on sets such as union and intersection of sets, complement and negation of a set.

Procedure:

(i) Name the sets involved in problems using capital alphabets

(ii) Observe the sets for common elements to be shown in the Venn diagram by intersection of sets

(iii) Observe the sets for only elements to be shown exclusively by individual sets.

(iv) Observe the sets for any elements in the problem but not present in any set to be shown in the Universal Set.

Set A represents the numbers (between 1 and 50) divisible by 3 and set B represents the numbers divisible by 5. In the corresponding Venn diagram, the region common to the two circles represents the set of numbers divisible by both. The Venn Diagram not only answers the numbers that are divisible by 3 or 5 or both but also numbers that are only divisible by 3 or 5. From the Venn diagram, we can list them.

$A_{3\ only}$= {3,6,9,12,18,21,24,27,33,36,39,42,48}

There are 13 numbers divisible by 3 only and

$B_{5\ only}$= {5, 10, 20, 25, 35, 40, 50}

There are 7 numbers divisible by 5 only

A ∩ B = {15, 30, 45}

There are 3 numbers divisible by 3 and 5 both. Thus; total number of positive integers between 1-50 (inclusive) divisible by 3 or 5 or both are 13+7+3=23.

This problem can be solved by using the principle of inclusion and exclusion cardinal number of sets also as discussed above:

n (A) =16, n (B) = 10, n (A ∩B) =3 using the relation, n (A U B) = n (A) + n (B) –n (A∩ B) = 16+10-3= 23.

Venn diagrams not only solve the problems involving pure mathematics such as discussed above but also real-world problems involving sets.

Lesson Plan 3

Topic: Egyptian Way of Writing a Fraction.

Standards: State Mathematics Education Standards:

- 2.1- Numbers, Number systems and Number Relationships. Types of Numbers (e.g.; whole, prime, irrational, complex). Equitant Forms (e.g.; fractions, decimals, percents.

Objectives:

- Students have to understand the Egyptian way of writing a fraction (i.e unit fractions)
- Students have to learn methods of how to write fractions into unit fractions.

Pre- Requite Knowledge:

Students have already become familiar with numerator, denominator, and reciprocal of a fraction. Even and odd numbers.

Time Schedule: Complete topic will be covered in three periods of fifty minutes each.

Procedure:

- Students are asked to write at least 5 fractions.
- Students are asked to write numerator and denominator of one of the fraction of their own choice,
- Students, who chose, even denominators, are told to split up the numerator as the sum of the factors of denominator.
- Reduce the fractions to lowest terms. For example: 11/16= 8/16 + 2/16+1/16= ½+1/8+ 1/16

Annotated Bibliography/ Resources:

Ramanujan's Notebooks: New York: springerverlag: 1994

Bruce C. Berndt (Author); Second Notebook, Journal of the Indian Mathematical Society, Master Theorem.

G.M. Hardy and E.M. Wright: Index to the classic text (4th/5th edition 1979). Hardy & Wright's The Theory of Numbers (published in 1938).

www.mathpropress.com/HardyAndWright.html

Campbell, Paul. "A 'Practical' Approach to Egyptian Fractions." Journal of Recreational Mathematics 10 (1977-1978) 81-86.

Burshtein, Nechemia. "On Distinct Unit Fractions whose sum equal 1," Discrete Mathematics 5 (1973) 201-206

Stewart, B. M. “Egyptian Fractions, “in Theory of Numbers, pp. 198-207.” The Macmillan Company 1964.

Edwards, A. W. F. Cogwheels of the Mind: The Story of Venn diagrams……………..

www.math. Cornell.edu/~henderson/biblio/-

Venn Diagrams- Adult Basic Education (ABE) The Real Number System; Venn Diagrams; Operational Words and Phrases; Trigonometric Ratios……………..

www.aeel.gov.sk.ca/evergreen/mathematics/…/index. Html.

Critique on Venn Diagram in Philosophy: Venn Diagram Basic…….contains examples of double cell and expanded Venn Diagrams…….

www.sciencecentral.com/…/critique on Venn diagram in philosophy.

APPENDICES- STANDARDS

Pennsylvania’s public schools shall teach, challenge, and support every student to realize his or her maximum potential and to acquire the knowledge and skills needed to:

2.1- Numbers, Number systems and Number Relationships. Types of Numbers (e.g.; whole, prime, irrational, complex). Equitant Forms (e.g.; fractions, decimals, percents.

2.1.8

A. Represent and use numbers in equivalent forms (e.g., integers, fractions, decimals, percents, exponents, scientific notation, and square roots)

B. Simplify numerical expressions involving exponents, scientific notation and using order of operations.

2.1.5:

A. Use expanded notation to represent whole numbers or decimals.

B. Apply number theory concepts to rename a number quantity.

C. Demonstrate that mathematical operations can represent a variety of problem situations.

D. Use models to represent fractions and decimals.

E: Explain the concepts of prime and composite.

G: Develop and apply number theory concepts (e.g.; primes, factors, multiples, composites) to represent numbers in various ways
2.1.11:

A. Use operations (e.g., opposite, reciprocal, absolute value, raising to a power, finding roots, finding logarithms).

2.2. COMPUTATION and ESTIMATION:

A. Develop and use computation concepts, operations and procedures with real numbers in problem- solving situations.

B. Use estimation to solve problems.

C. Construct and apply mathematical models (e.g., Venn diagrams, graphs, conceptual maps etc.)