Probabilities in the Poker Game – Curriculum Unit

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Overview

Students gain a deeper understanding of mathematical concepts if they have the opportunity to explore real-world applications. The abundance of game shows, casinos, and lotteries demonstrates how enjoyable and exciting games can be. During this unit, students will have the opportunity to study the applications of probabilities in the poker game and have fun at the same time. Different examples involving the counting principle, permutations, and combinations will be studied but the focus will be on investigating and interpreting probabilities for different types of 5-card poker hands: a pair, two pairs, full house, four cards of a kind, royal flush, and straight flush. My challenge is to create an interesting and in-depth activity, balance the theory with practical use cases, and maintain an appropriate pace of effort.

Rationale

As a teacher at Randolph Career Academy, I have the challenge of making mathematics meaningful to my students. Most of the students are performing poorly at standardized tests, but in whatever way the state or district tries to identify or label the problems at our school, the students prove to have logical thinking skills and complex understanding capabilities. They have not, however, found mathematics to be fun and exciting. This unit focuses on studying different outcomes in the act of playing poker as a means of learning mathematics and having fun at the same time.

Objectives

The curriculum unit purpose is to help Algebra 2 students to apply probabilities to real life situations, make valid predictions on poker hands, analyze their predictions and use new knowledge to defend them or to create new ones. Students' critical thinking, reasoning and problem solving skills are expected to be enhanced by studying this unit. The students should understand during the unit that poker could be not only a game, but also a means to grasp difficult concepts of combinations and probabilities.

Strategies

In order to build students' mathematical knowledge as well as providing helpful skills, I propose a multi-day project-based unit that will involve the students in a process of learning combinations and probabilities through the study of poker game. Brainstorming instructional strategies will be implemented. Students will be involved in inquiry-based activities, along with exposure to multimedia products and hands-on experiences.

Several scenes involving poker playing from the Maverick movie (1994, Mel Gibson, Jodie Foster) will be presented to the students. They will use information from some Maverick movie scenes and hands-on activities to connect mathematical concepts with real-world problem settings.

Alignment with NCTM standards

In accordance with NCTM Standards for Mathematics Curriculum at the 9-12 level, students will:

- Apply the process of mathematical modeling to real-world problem situations.

- Reflect upon and clarify their thinking about mathematical ideas.

- Express mathematical ideas orally and in writing.

- Construct simple arguments and judge their validity.

Lesson1: The Counting Principle

If there are two events E_1 and E_2 where the first can happen in n_1 different ways and the second in n_2 different ways, then together the events can occur in n_1 x n_2 different ways, assuming that the events are not influencing each other.

This generalizes to k events E_1, E_2, …, E_k with the number of possibilities for the corresponding events n_1, n_2, …, n_k. The total number of possibilities is n_1 x n_2 x …x n_k.

Let's suppose we select new uniforms for a team. The pants are coming in 2 styles, shirts in 3 styles, and hats in 4 styles. In how many different ways can a 3-piece uniform be selected? To determine how many choices there are in total, make a tree diagram, which will show 24 branches. Thus, there are 2 x 3 x 4 choices in all.

How many combinations exist for a lock that opens with a sequence a 3 numbers from 1 to 40? The total number is 40 x 40 x 40 = 64,000.

Exercises:

1. A company is setting up phone numbers for a new town. Each number must have 7 digits and must not start with 0 or 1. How many different numbers are possible? Repetition of digits is allowed.
2. Suppose a student is given an exam on which there are 16 questions. The first five items have 4 choices each, the next five have 3 choices each, and the last six are true or false. If the student answers items 3, 6 and 10 correctly and guesses all the others, how many different ways can he complete the test?
3. Next year you are taking math, English, history, chemistry, physics, Spanish and physical education. Each class is offered during each of the seven periods in the day. In how many different orders can you schedule your classes? Let's suppose your best friend is taking three of the seven classes you are taking. In how many ways can you schedule your classes so that you and your friend share four classes? Assume that each of the non-shared classes is offered during each of the periods in the day.

Lesson 2: Permutations

A permutation of some or all of the elements of a set is any arrangement of the elements in definite order. For example, for the set {a, b, c} there are 6 permutations: abc, acb, bac, bca, cab, cba.

To find the number of permutations without listing them, the fundamental counting principle can be used:

- There are 3 ways to fill the first position.
- For each way to fill the first position there are 2 ways to fill the second position.
- For each way to fill the first and the second positions there is only 1 way to fill the third position.

In this way, the number of permutations of the elements of {a, b, c} is 3 x 2 x 1 = 3! = 6.

Exercises:

1. The automobile license plates need 3 letters and 4 digits. How many unique codes are there under this scheme?
2. Suppose we have to arrange 8 students in a row. In how many ways can you do this?
3. How many permutations of the letters in the words BUBBLE and REARRANGMENT are there?
4. If a class has 24 students, find the number of permutations for each situation: 6 students for a basketball team, 10 students for the mock trial team, 1 student for the president, and first, second, and third place in the art show.

Lesson 3: Combinations

A combination is a way of choosing k objects out of a collection of n, or is a subset of k objects from a set of n. The number of ways of choosing k objects out of n is designated $C(n, k) = {}_nC_k$ and is usually read as "n choose k".

$${}_nC_k = C(n, k) = P(n, k) / k! = n! / ((n-k)! \times k!)$$

When you count combinations of elements of a set, the order in which they are listed is disregarded.

Let's find out how many lines are determined by 7 points, no 3 of which are collinear. Two points determine a line, therefore n = 7 and k = 2: ${}_7C_2 = 7! / (2! \times 5!) = 21$.

In the game of poker a hand consists of five cards dealt from a deck of 52. How many different poker hands are there? We start out by considering permutations of 5 out of 52.

$$P(52, 5) = 52 \times 51 \times 50 \times 49 \times 48$$

Each hand will be counted more than once. How many times will each hand be counted? A given hand of 5 cards can be arranged in 5! = 5 x 4 x 3 x 2 x 1 = 120 different ways, so the total number of hands is

$$52 \times 51 \times 50 \times 49 \times 48 / 120 = 2{,}598{,}960.$$

Exercises:

1. How many diagonals can be drawn in a 76-side convex polygon?
2. There are 10 points in a plane, no 3 of which are collinear. How many different polygons can be drawn using these points as vertices?
3. The summer Olympics games had 16 countries qualified to compete in soccer. How many different ways can teams of 2 be selected?
4. A state lottery game contains numbered balls numbered 1 through 40. Five balls are drawn randomly. If a person matches all 5 numbers they win the jackpot.

a) How many possible number sequences are there if the numbers must be matched in the order that they were drawn?
b) How many possible number sequences are there if the numbers can be drawn in any order?
c) Find the probability, if the order does not matter, that any single number combination is drawn.

Lessons 4-5: Probabilities in the Poker Hand - Problem Set

Poker is a game that is played all over the world. There are many forms of poker, but they differ only in minor details and all follow the same basic principles. In general, the players use a standard deck of fifty-two playing cards. Certain five card combinations are recognized in all forms of poker and the ranking of these combinations gives a poker hand its strength.

The 52 cards are divided into 4 suits - clubs, diamonds, hearts and spades – and each suit is divided into 13 ranks: 2, 3, 4, 5, 6, 7, 8, 9, 10, Jack, Queen, King, and Ace. In the game of poker a hand usually consists of 5 cards drawn from a standard deck of 52 cards.

Materials:

- Enough sets of card so that the class can be divided up into groups of four
- Graphing calculators
- Worksheets (attached)

Guided Practice:

Students should set up the cards and discuss all possible hands of five cards they could get during a poker game. The teacher will observe the interaction of students during the project, check for understanding and consider all different strategies they used for making and defend/change predictions. The information recorded by the students on the worksheets will be checked.

The types of winning hand from highest to lowest are:

1. **Royal Flush** – consists of a ten, jack, queen, king and ace from the same suit.
2. **Straight Flush** – five cards that are both straight and a flush
3. **Four of a kind** – four cards of the same rank
4. **Full House** – 3 cards of the same rank and two others of the same rank
5. **Flush** – five cards in the same suit
6. **Straight** – five cards that can be arranged so that each card has rank 1 greater than previous card
7. **3 of a kind** – 3 cards of the same rank and two others not of the same rank

8. **Two pairs** – 2 cards of one rank and two cards of another rank
9. **Single pair** – two cards of same rank and none of the other 3 cards of same rank.

Solutions:

1. **Royal Flush** – consists of a ten, jack, queen, king and ace from the same suit. There are only 4 hands of this kind. The probability to get a royal flush is 4/2,598,960 = 0.00000153908.
2. **Straight Flush** – five cards that are both straight and a flush. The cards are arranged so that each one has a rank 1 higher than the previous card. The Ace can be used as the lowest and as the highest-ranking card, so there are 10 possible straights in a suit (ex: 6, 7, 8, 9, 10). For 4 suits, there are 4 x 10 = 40 straights flushes, but if we exclude the royal flushes, the number of hands will be 40 – 4 = 36. Because the total number of hands of 5 cards is ${}_{52}C_5$ = 2,598,960, the probability to get a straight flush is 36/2,598,960 = 0.0000138517.
3. **Four of a Kind** – this hand has the pattern AAAAB, where A and B are two cards of different kind. There are 13 possibilities or ${}_{13}C_1$ for card A rank and ${}_{12}C_1$ = 12 possibilities for card B rank. Because we need 4 cards of one specific rank (card A), we can find out that there are ${}_4C_4$ = 1. To fill in the last card, there are ${}_4C_1$ = 4 ways to choose it. In total, there are 13 x 12 x 4 x 1 = 624 possible ways of having hands with 5 cards, four of which have the same rank. The probability to get a hand of this kind is 624/2,598,960 = 0.000240.
4. **Full House** – this hand has the pattern AAABB, where A and B are two cards of different kind. For 3 of a kind (AAA), there are ${}_{13}C_1$ = 13 ways of choosing the rank. To choose 3 cards of that rank, there are ${}_4C_3$ = 4 ways to do that. For 2 of a kind (BB), there are ${}_{12}C_1$ = 12 ways of choosing the rank and ${}_4C_2$ = 6 ways to have 2 cards of that rank. The number of such hands is 13 x 4 x 12 x 6 = 3744, so the probability to get one of these is 3744/2,598,960 = 0.001441.
5. **Flush** – this hand includes 5 cards from the same suit, excluding the straight flushes. The number of hands will be (${}_{13}C_5$)(${}_4C_1$) – 40 = 5108, with probability approximately 0.0019654.
6. **Straight** – this hand includes 5 cards in a sequence where each card has rank 1 greater than previous card, and the cards allowed to be from same suit or different suits. The number of hands of this kind is 10 x (${}_4C_1$) x (${}_4C_1$) x (${}_4C_1$) x (${}_4C_1$) x (${}_4C_1$)

= 10,240, with probability 0.003940. If straight flushes and royal flushes are excluded, the number of hands will be 10,200, with probability 0.00392465.

7. **Triple** (AAABC) – in this case the hand should contain 3 different ranks, for cards A, B, respectively C. Looking at one suit, the card rank for the triple AAA can be found as ${}_{13}C_1$ = 13 and the ranks for the other two cards BC is ${}_{12}C_2$ (the order of the ranks is not important here). For a rank of card A, there are ${}_4C_3$ = 4 triples, and for chosen ranks of cards B and C, there is the same number of ways of choosing card B, respectively card C : ${}_4C_1$= 4 ways. The number of hands will be $({}_{13}C_1)$ $({}_4C_3)$ $({}_{12}C_2)$ $({}_4C_1)$ $({}_4C_1)$ = 54,912. The probability to get this kind of hand is 0.021128.
8. **Two pairs** (AABBC) – since for the group AABB the order of ranks is not important, the number of ways of choosing 2 ranks is ${}_{13}C_2$. The card C has ${}_{11}C_1$ = 11 options for the rank. The groups AA and BB can be each chosen in ${}_4C_2$ = 6 different ways for a given rank, and card C can be chosen in ${}_4C_1$ = 4 ways. The total number of hands is $({}_{13}C_2)$ $({}_4C_1)$ $({}_4C_2)$ $({}_{11}C_1)$$({}_4C_1)$ = 123,552 and the probability for having a hand of this kind is 0.047539.
9. **One pair** (AABCD) – there are 4 different ranks in this hand. To choose the rank for the pair, we have ${}_{13}C_1$ = 13 ways to do it. To find the ranks for cards B, C, and D, we have to consider that the order for their ranks is not important, so there are ${}_{12}C_3$ ways. For given ranks, pair AA should come in ${}_4C_2$ = 6 ways and cards B, C or D in ${}_4C_1$ = 4 ways each. There are $({}_{13}C_1)$ $({}_4C_2)$ $({}_{12}C_3)$ $({}_4C_1)$$({}_4C_1)$$({}_4C_1)$ = 1,098,240. The chances of getting a one-pair hand are 1,098,240/2,598,960 =0.422569 or about 42%.

Worksheet

Hand Type	Total Number of Hands	Probability
None the same ABCDE		
One pair AABCD		
Two pairs AABBC		
Three of a kind AAABC		
Straight (ex: 5 6 7 8 9)		
Flush (five cards from the same suit)		
Full House AAABB		
Four of a kind AAAAB		
Straight Flush		
Royal Flush (A K Q J 10 from the same suit)		

Assessment:

When manipulatives for the classroom are devised, assessment is not always very precise. During this activity, the teacher can judge if the students are staying on task. It would be good to have the students take a matching quiz before and after finding the number of combinations and probabilities for each kind of hand. The rubric (see the attached document) should be used as a guideline for both teachers and students to understand that there will be a grade given to each day of work. The rubric is generic in nature and modifications should be made for each type of project.

Exercises:

1. Calculate the same probabilities and ranking given that a single joker card, which can stand for any hand and any suit, is added to the deck. Five of a kind is not taken into consideration. Explain whether or not adding a joker card changes the relative rankings of the hands.
2. Suppose we have to make a committee of 7 people. There are 25 families (husband and wife) to choose from, but only one member from each family can be chosen. How many different committees can be made?

Bibliography:

1. Schultz, Ellis, Hollowell, Kennedy, Engelbrecht, (2004), Algebra 2, Holt Rinehart, and Winston
2. Larson, Kanold, Stiff (1993), Algebra 2, D.C. Heath and Company
3. Bettye C. Hall, Mona Fabricant (1990), Algebra 2 with Trigonometry, Prentice Hall
4. www.math.hawaii.edu/~ramsey/Probability/**PokerHands**.html
5. www.chemistry.ohio-state.edu/~parker/**poker**.html

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RUBRIC: Group Project - Probabilities in the Poker Game

	1 = Poor	2 = Average	3 = Good	4 = Excellent
Contribution to Group's Tasks and Completion of Own Task	• Chooses not to participate • Shows no concern for goals • Does less work than others	• Participates inconsistently in group • Shows some concern for goals • Does almost as much work as others	• Participates in group • Shows concern for goals • Does an equal share of the work	• Participates actively in group activities • Helps direct group in meeting goals • Does a full share of the work or more
Discussion Skills and Active Listening	• Does not participate in group discussions • Seems to be bored with conversations about the project	• Shares ideas occasionally when encouraged • Listens to others and on some occasions makes suggestions	• Shares ideas when encouraged • Listens to others and participates in discussions consistently	• Provides many ideas related to the project • Listens attentively to others
Communication	• Never speaks up to express excitement and/or frustration • Is openly rude when giving feedback • Refuses to listen to feedback	• Rarely expresses feelings and/or preferences • Sometimes hurts feelings of others with feedback • Argues own point of view over feedback	• Usually shares feelings and thoughts with classmates • Gives feedback in ways that do not offend • Accepts feedback reluctantly	• Clearly communicates ideas, personal needs and feelings • Gives feedback to others without offending • Accepts feedback from others willingly
Time	• Some work never gets completed	• Work is late but is completed in time to be graded	• Work is ready very close to the agreed time	• Work is ready on time or ahead of time
Project Quality	• Major elements of the project are incomplete or missing, demonstrating a minimal understanding of the concepts and procedures.	• Conclusions and explanations demonstrate a partial understanding of the concepts and procedures required by the task.	• Conclusions and explanations are complete and correct. Some minor errors or omissions are accepted.	• Conclusions are accurate and thorough. A minor omission is acceptable.