Understanding Probability

Elizabeth Gourlis
William C. Longstreth Elementary

Overview

This unit was designed to reinforce the current standards-based curriculum for elementary education in Philadelphia Public Schools. The School District of Philadelphia's math curriculum uses Math in Context. Math in Context (MIC) is a program for middle year classes that shows students real world math problems and then discusses the abstract concept behind those applications. This MIC program introduces many math domains for the students to become familiar with and explore rather than understanding and being introduced to one domain at a time. This unit was created to enhance the MIC model Philadelphia teachers need to teach. This unit will give teachers and students time to explore the unit of Probability. The unit's activities are the hands on component that MIC is lacking. They will enhance the quick overview MIC gives and create in-depth understanding and comprehension of the Probability unit. The students will create personal as well as peer understanding through group work, class discussions and projects that will physically show probability. Through the class discussion and group work, students will connect abstract ideas and concepts of probability and be able to aid classmates that are below grade level.

This curriculum will highlight problem solving, communication, connections, reasoning and proof as well as mathematical representations with a focus on African American learning styles. The intended learners are sixth grade low-income inner city students. The obstacle to over come is the standards based learning ideal that is a one size fits all program. The prescribed School District of Philadelphia curriculum's lessons do not account for culturally relevant learning, below grade level learners, IEP considerations or classroom disruptions. Many of the lessons need to be adapted and adapted again to engage the disenchanted learner. This curriculum is an attempt to alter the math landscape for a predominately African American classroom. By using the research and data provided, teachers can adapt their lessons times for the maximum benefit of their inner city classrooms here in Philadelphia.

This unit has low-ability, high enthusiasm students in mind. These particular students are below grade level in reading and far below grade level in basic number sense (multiplication tables, skip counting, and division). The students enjoy math however limited their understanding. This unit will engage them with high interest subject matter (fashion, hands on work, etc.) that will create a bridge to the math concepts needed to understand probability. The students will be able to incorporate their own ideas into mathematical learning which will increase their retention rate.

The unit's goals are to teach from the bottom up the probability domain while showcasing to students there is more than one way to achieve understanding (and ultimately for test purposes-the right answer) in math. Students will have multiple options during this unit to find the probability of an event. Students will have discussion time, flexible grouping, technology pieces and pen and paper work to cover the range of learning styles that a teacher encounters in a classroom.

The approach to learning probability will have options that will range from formulaic equations, matching opportunities, record keeping and oral communication. Allowing these students to choose a solution will give them more autonomy over their learning and play to their learning strengths. Many other probability units attempt to quickly move children to the formula of probability. This unit is designed to start with the idea the students know very little about probability and build from the ground up. There are many repeated activities to allow students to achieve an end results without teacher led activities or discussion. Students that understand the concepts quickly can feel empowered to aid fellow students. The activities can be combined or eliminated if the concept is grasped.

My attempt with this unit is to create a mathematical literate classroom that fosters a hands-on approach to understanding probability. Incorporating interactive activities, designated vocabulary time, a slow pace and flexible grouping will foster safe environment to raise below grade level students as well as grade level students without alienation and ridicule. The activities are varied to allow students that need more time and understanding to have more time as well as challenging the students that are ready to move on.

Rationale

Our world is very uncertain. Will it rain? Who will win the basketball game or will someone get into fight today? We rely on the mathematics of probability to find our answer to every day questions. Yet students understand little about the terms used in probability such as certainty, likely, unlikely and or impossible. My students struggle with the notion of one part of chance is connection to another part of chance. For example, if there is sixty percent chance of rain, does that mean rain is likely, certain or unlikely. If it might rain, the weather also might be sunny or cloudy. Discovering how to

teach probability as well as creating interactive hands-on activities will create space for students to delve into probability and chance.

Understanding how and when to lay the groundwork foundation of probability will allow teachers to work smarter not harder. Piaget (1950) and Piaget and Inhelder (1974) believed children were not born with the notion of chance. Children needed to be taught the skills to comprehend probability. Teachers should be aware of the timing to teaching probability so the cultivation of understanding is appropriate.

There are three stages toward maintaining and expanding the understanding of probability throughout elementary school. This awareness will create prior knowledge that will be built upon during the elementary, middle and high school years. These stages are based on Piaget's research into how children learn. The first stage is called pre-operational. Students are in pre-kindergarten to first grade within this stage. This stage is when children lack the intellectual awareness to recognize when an event is chance and when it is not chance. There is also a lack of logic within this stage of development. Students do not have rational thought within this time frame. The child links together unrelated events, believes certain objects are alive, can not comprehend point of view, and cannot reverse operations. This stage should showcase tangible objects and a good use of questioning to distinguish those objects.

The second stage is concrete operational. This stage is from first grade to fifth grade. This stage is when children can tell only absolute differences all of the possible outcomes. Students at this stage use their senses in order to understand their immediate world. Students can list some of the outcomes to an event but not all.

The third stage is formal. This stage is from age eleven years and up, which can be fifth grade and up. The children can create combinations, permutations and proportions. Students can quantify simple events at this stage.

One of the important challenges in mathematics teaching is to help students make connections between the mathematics concepts and the activity. By allowing students to "take away" products and ideas after the lesson is over is the surest way to create lasting success in classrooms that often to do not succeed.

Bearing these notions in mind, teaching children should follow the progressive stages of understanding probability. The following curriculum and activities will be designed upon the idea that stage one understanding needs to be achieved before stage two understanding can be started. Finally with stage three understanding that will build and solidify probability.

A large part of this sixth grade math curriculum in the School District of Philadelphia is discussions. Students will discuss prior knowledge and cultural understandings on mathematical activities of probability. By sharing conversations, students will learn to

rely on each other, tolerate each other and work together. These are some issues inside inner city classrooms that stop the learning and continue the fighting, bickering and bullying that is plaguing our schools. African American students learn a lot by discussion, more so than their Caucasian counterparts. Setting time aside within the lesson for conversation, this will allow students to feel heard and be able to move on to the next section of the lesson. By allowing a part of the outside world into the classroom, there will be space for the classroom in the outside world.

The goal of this prospectus is to aid students to make sense of their world using data, understand variability and quantify uncertainty and to draw valid conclusions from data as well as question data collection. The teacher will aid students in deconstructing misconceptions. By discussing prior knowledge, removing the misconception and replacing it with a solid definition/answer/activity, students will progress through the levels of learning and understand how to learn/ask questions.

Mathematical Background:

The flexible grouping within this unit will give students the confidence that they can be in charge of their own personal learning. The shared belief that students can rely on a peer for under standing or aid another student in mastery is a highlight in this unit. Teachers learning strength may not be students'. There fore using all aspects of the classroom to teach will magnify in depth learning and retention.
Student led learning ensure far greater results than teacher led learning. Using this lesson as a stepping-stone, students can take the lesson of probability as well as the journey of learning to the next standard, unit or domain and continue success at their new level of understanding. Students should be able to use the hands on approach in other math lessons and feel confident in that method.

Students will by no means magically become grade level proficient but with understanding that they are in charge of their own learning, students should feel empowered in their classroom to use the resources within to support what the teacher is doing. Inner city students that are pushed through grades feel disenchanted with the learning process, by create a student lead unit, a teacher can foster buy-in for those ambitious students that want to understand but may not be able to convey or even understand their mathematical needs.

With the current climate of accountability in major school system, there is a push to streamline students' understandings. This overt attention to teaching toward the test leaves out time teacher's need to create multi-faceted multi-dimensional lessons. Administrators and teachers forget that achievement in the classroom happens when student are engaged and captivated by the learning.

The traditional notion of intelligence ignores the children that might be linguistic, logical-mathematical, or visual-spatial learners. The traditional perception that learning comes

from listening or watching is not in tune with the culture of learning outside the classroom. The subject math lends itself to hands on learning. Using manipulatives, writing out problems, drawing the images among other things gives students of all learning types the chance to excel. Math teachers can not forget to use the tools of math in all its forms to create a bond with student and subject.

Inner city students have a relationship with education that is different than their suburban counterparts. Before even stepping into the classroom, the divide can be noticed. The cost per pupil, the environmental noise, stress and overall scenery that surround an inner city student are less conductive to learning. Inner city teachers and students deal with more issues in the classroom that do not pertain to the actual teaching of lesson. The inner city teacher needs to be aware of these distractions to allow time and space for them as they create lessons that keep their students engaged and interested.

This curriculum is geared toward the dynamic fluid connection between the students and teacher. There will be set time to reflect, within cooperative groups or alone, and a freedom to critique the mathematical concepts as well. Boykin (1983) identified nine cultural dimensions that might be considered unique among African Americans: spirituality, movement, verve, affect, harmony, communalism, expressive individualism, orality and social time. Boykin researched communalism, verve and movement with low-income African American students, though without a mathematical focus (2000). Spending any amount of time within a math classroom, there is very little verve, communalism or movement involved. To assure success in our classrooms, we need to be cognizant of our students, their cultures and learning styles.

In other research on African American cultural inclination, Professor James M. Jones research created a major area of work centered on the idea of TRIOS. This is a psychological worldview that combines African inspired processes of Time, Rhythm, Improvisation, Orality and Spiritually instead of a Eurocentric focus. Through these pieces (TRIOS): time meaning event time rather than clock time-(participating in an event rather than keeping an eye on the clock), rhythm meaning noticing an internal response to the patterns of the external world (a psychological experience of recurring patterns of behavior), improvisation is a way to bridge the internal with the external realms (creative solutions to problems), orality meaning the oral tradition of African Americans (students wanted a face to face communication) and finally spirituality a central aspect of African origins(belief that a higher power influences us).

Bearing Jones and Boykin in mind as inner city teachers create curriculum for their students will enhance and deepen the core concepts that need to be taught and understood. For example planning time into the lesson for the event (concept/standard) to be completed not just during the time allotted math lesson will aid students in being able to understand and work the problem. Using groups to work out the answer to a problem and sharing those results with the class is another aspect of using cultural inclinations to enhance education.

Using cooperative learning and flexible grouping requires students to understand that the group aspect of learning is not gossip time. The group time is for discussion of ideas, giving help and having the allotted space to think about the math. Flexible grouping gives a teacher an opportunity to spread out the classroom knowledge. Students that need extra help give other students opportunities to teach what they know. Students that grasp concepts quicker are perfect to pair with students that need extra help. This dynamic is a version of differentiate instruction that will aid the teacher in moving the lesson along and not needing to re-teach specific sections.

What to expect:

Mastering vocabulary is like introducing the pre-operational stage for sixth grade. This section can take 3 days or longer depending on the students' ability to grasp the concepts. Rushing through the vocabulary is not recommended. Eliminate an activity if pressed for time. Introducing vocabulary terms: certain, likely, unlikely, or impossible, independent event, dependent event, random.

Students might take a very long time in creating their various outfit components. This part could be done for homework the night before or after a morning work. Students might also lose pieces of their outfits. A suggestion is to for the teacher to have extra of each component: shirts, socks, shoes, marbles, etc.

Things you might encounter:

With flexible grouping a teacher could encounter poor grouping where students are talkative and off task. Another group could be mismatched with students that grasp a concept easily or with difficulty. A group could have one student is teaching others without adding to their understanding or learning.

Flexible grouping when the group climate is balanced is very beneficial; students learn tolerance, how to work cooperatively and most importantly, students take ownership of their own thinking/learning. Students will be less inhibited by teacher led discussions to share their strategies within the comforts of the student group.

Students might lose track of time when creating a combinations or flipping the coins. There need to be set time limits and check-ins with students to keep the focus and attention to the math work.

This unit uses technology in the warm ups. This may not be helpful for some classrooms that are not equipped to show videos. The videos are worth watching for the teacher.

Strategies

A large portion of the student body does not have basic mathematical skills and terms. To aid students in remember the terms of probability, this unit has attempted to incorporate various learning styles. By differentiating the learning medium from paper and pen to cuts outs to video, the students will have touched the term and its definition multiple times. This repetition will allow students more ways to absorb understanding and ensure success. The current curriculum is fast paced and does not account the low performing student's need to grasp a concept or definition first before participating in an activity.

Another strategy is data collection. Students will be recording their combinations, coin tosses etc. A t-chart graphic organizer allow students to record the results and write notes to the side about what if any patterns they notice and/or comments they might have about the data.

If the classroom has computers and students are familiar with Excel before this lesson, an Excel spreadsheet will allow students to record data in a more scientific and serious minded manner. I would not recommend using Excel if the students are not previously familiar with the program.

Use formulas or use manipulatives to solve the problem. Either strategy will achieve the answer, but students will be able to choose the right one for them. Using manipulatives allows tactile students to grasp the concept and answer while more analytical students will gravitate toward the formula to solve the problem.

Classroom Activities

Day 1: recommended time 90 minutes

Objective: Students will grasp the definition of the following vocabulary terms: certain, likely, impossible, unlikely in relation to chance events.

Warm up: Write the term certain and impossible, likely and unlikely on the board. Give students time in their desk group or with partners to brainstorm the definitions. Some questions to elicit definitions: what does likely mean? What does certain mean? What is impossible? What is unlikely to happen today? If something is certain, can it be more than one thing? What is an example of something that is likely to happen? What is an example of something that is certain to happen?

Whole class discussion/practice: If the classroom is set up for technology the first video is an interactive video using a voice identified as Ms. Bishop. Ms. Bishop discusses the terms in both videos with examples how the vocabulary applies to probability. In each video Ms. Bishop discusses impossible, certain, likely and likely. Both videos allow time

for the students to answer and process the questions. Allot time for questions and responses after the video.

http://player.discoveryeducation.com/index.cfm?guidAssetId=7FAEAEF2-6132-47B5-89A9-EA033FD0E1E4&blnFromSearch=1&productcode=US

http://player.discoveryeducation.com/index.cfm?guidAssetId=0766CEC3-A56C-48A2-B110-C6A438C3051F&blnFromSearch=1&productcode=US

Individual Guided Practice/Group Work: To check for understanding in relation to the vocabulary, students will use weather maps to correctly define the terms: certain, likely, unlikely, or impossible. If the classroom is technology ready, use weather.com or accuweather.com to obtain weather maps. Using the radar section, have the students predict the weather. The students should create mini forecasts statements using the vocabulary terms: certain, likely, unlikely, or impossible and a factual based weather prediction. For example: Tomorrow's weather is likely to be sunny due to the lack of cloud cover. Students should create simple statements based on the facts they are observing. This can be done with the newspaper as well. The weather section can be photocopied if there are not enough newspapers.

Discuss/Wrap up: Students should share out their weather forecasts. The meteorologist should explain the vocabulary term and why it applies to that forecast. As the class listens, they are checking to make sure the "meteorologist" is correct in their forecast. Students should share out comments or questions about the activity.

Homework: Students should continue writing the weather forecast for the week for their home city. Students can choose to a map to read and create their probable forecast for the next 5 school days.

Day 2: Recommended time 90 minutes

Objective: Perform simulations with concrete devices (e.g., dice, spinner) to predict the chance of an event occurring.

Warm up: Write the terms unlikely and impossible, likely and certain on the board and have students write the definitions or examples from what they remember from the previous day. Once again, if the classroom is equipped with technology to show a video, the following video showcases a teacher in the 3rd grade and her introduction to probability. Technology is just another tool for children to see, hear and understand the terms. The following video is from the School District of Philadelphia's School Net Page: United Streaming Discovery Education. This video is 14:39 minutes long and titled: Introducing Probability-segment of: Mathline: Elementary School Math Project: Program 08 Discuss what the students heard and saw. Discuss if the video clarified some lingering

questions. Did the video make sense?

Materials: Print out attached shirts, pants, shoes and socks (homework) sheets for students to cut out and color in. Make sure each student has 2 brown paper bags or opaque container to put cut-out shirt and pants into separately

Whole class discussion/practice: Start a discussion about how the students decide what they are going wear in the morning. Would it be possible that they would wear a winter coat with shorts? Could it be possible to wear red sneakers and green pants? The students are going to have an opportunity to create outfits that are possible to wear. Write the following articles of clothes in a straight line across the board: jeans, green pants and khaki pants. Below the line of pants, write blue shirt, green shirt, yellow shirt and purple shirt. The students will have a set of shirt and pants to color, then put into an opaque container. Their job will be to reach into the shirt bag then the pant bag and record.

Individual/Group Guided Practice: Students will create their own outfits and record the combination-regardless of what they think of the match. They will need 4 of each pant color and 3 of each shirt color. The students need to pair the pants with all shirts. Students should share out comments or questions about the activity.

Wrap up/discussion: Ask the students to share out the total number of combination they had regardless of what their opinion of the outfit. Have students share out their number of outfits and color combinations. After hearing from the students, discuss that they should have 12 outfits. The four shirts need to match with the three pants. The blue jeans can be paired with all 4 shirts, the green pants can be paired with all 4 shirt and the khaki pants can be paired with all 4 shirts. Four plus four plus four equals twelve or 4 x 3 = 12.

Homework:
Choose an outfit: 2 pairs of shoes- (for example Chuck Taylors and flip-flops). Use attached images for students to decorate their shoes and socks. The students will be pairing socks with the shoes. Students will record the outfits as they are created. Students will need to share out the number of combinations made and be ready to discuss if the combinations are wearable in public.

Day 3: Recommended 90 minutes

Objective: List the outcomes and calculate the probability of a simple event.

Warm up: Write the terms independent event and dependent event. If the classroom is set up for technology, the following video is 1:44 minutes long and introduces the idea of independent and dependent events. The following video from the School District of Philadelphia's School Net Page: United Streaming: Discovery Education Independent Events-Ultimate Frisbee segment. Discovering Math (Audio Description Version):

Grades 09-12: Probability. Discuss what the students heard in the video. What other things can be considered independent (the numbered ball draw in the lottery)? What other things can be considered dependent (a team winning the game)?

Whole Class Discussion: Ask the students if they have ever played bingo. Does the other player's bingo board affect you chance of winning? Does your bingo board affect someone else's odds of winning? The bingo boards and a coin toss from the video are considered independent events. Ask the students if they would consider wearing green pants with a purple shirt in public. Some might not want to be seen with that combination on. The decision to not match green pants with a purple shirt is considered a dependent event. The pant and shirt combination do rely on each other. For students to observe what an independent event is, a coin toss will help. Discuss when a coin is tossed, there are two outcomes-heads or tails. There is half a chance the toss will be heads and half a chance the toss will be tails. This is a 50/50 situation. As students to predict the first 5 tosses on a sheet of paper. As the students if the coin tossed is heads, how likely will the next toss be tails? Highlight the notion, that each side of the coin has a 50/50 chance of being true. Then toss the coin five times and record the results. Ask the students about the accuracy of their predictions. Ask if there is any way to predict how the coin will be fall. When discussing the student's responses, highlight the idea that the outcome -heads or tails- is not contingent on the other side to produce the outcome.

Materials: coins of any denomination for individual or group work

Individual/Group Guided Practice: Students will predict what results could occur in 25 coin flips. Then the students will flip the coin 25 times and record the results. Students should share out comments or questions about the activity.

Wrap up/Discussion: Students will share out their results. Discuss the idea heads does not necessarily follow tails and vice versa.

Day 4: Recommended 90 minutes

Materials: marble sheet and a brown bag or opaque container.

Warm up: Discuss that drawing specific marbles out of a bag is a dependent event. The probability of drawing marbles out depends on how many marbles are in the bag.

Whole Group discussion: A jar contains 22 yellow, 26 green, 17 red, and 20 purple marbles. Use the attached sheet to color specific amount of marbles (22 yellow, 26 green 17 red and 20 purple marbles). Students should color all the marbles and place them into a brown paper bag or another opaque container. As students place marbles in the bag, they should total the marbles. Students should be aware that the marbles are also called

outcomes. Have students record the color of each marble they draw out. Students should work together to draw marbles out until all the green marbles are out of the container. Student should recognize and discuss the data. What do they notice? Students should discuss their ideas. Record their answers. Can they fill in the fraction below? What will their answer be to the following question: What is the probability a green marble will be drawn at random (P green)?

P(event) =getting a green marble $\frac{\text{\# of green marbles}}{\text{\# of total marbles}}$

Individual/Group Practice: Students should continue the whole group discussion at their seats. A jar contains 24 black, 11 orange, 10 brown, and 9 green marbles. Use the attached sheet to color specific amount of marbles (24 black, 11 orange, 10 brown and 9 green marbles). Students should color all the marbles and place them into a brown paper bag or another opaque container. As students place marbles in the bag, they should total the marbles. Students should be aware that the marbles are also called outcomes. Have students record the color of each marble they draw out. Students should work together to draw marbles out until all the green marbles are out of the container.

Student should recognize and discuss the data. What do they notice? They should discuss their ideas. Record their answers. Can they fill in the fraction below? What will their answer be to the following question: What is the probability a black marble will be drawn at random (P black)? Students should share out comments or questions about the activity. This is the first introduction of a formula for this unit. The students should note the formula whether they plan to use it or not. Each student should use the formula for at least one problem for homework if not more.

What is the probability a black marble is drawn at random (P black)?

P(event) = getting a black marble $\frac{\text{\# of black marbles}}{\text{\# of total marbles}}$

Wrap up/Discussion: Discuss the results. Check in with students about the formula and drawing marbles out. What did the students notice? How different was drawing the marbles out than creating combinations/outfits? Did the formula help or was it easier to see the drawing out of marbles.

Homework: A jar contains 4 pink, 11 green, 19 blue, and 6 violet marbles. What is the probability a violet marble is drawn at random?

P(event) = $\frac{\text{\# of pink marbles}}{\text{\# of total marbles}}$

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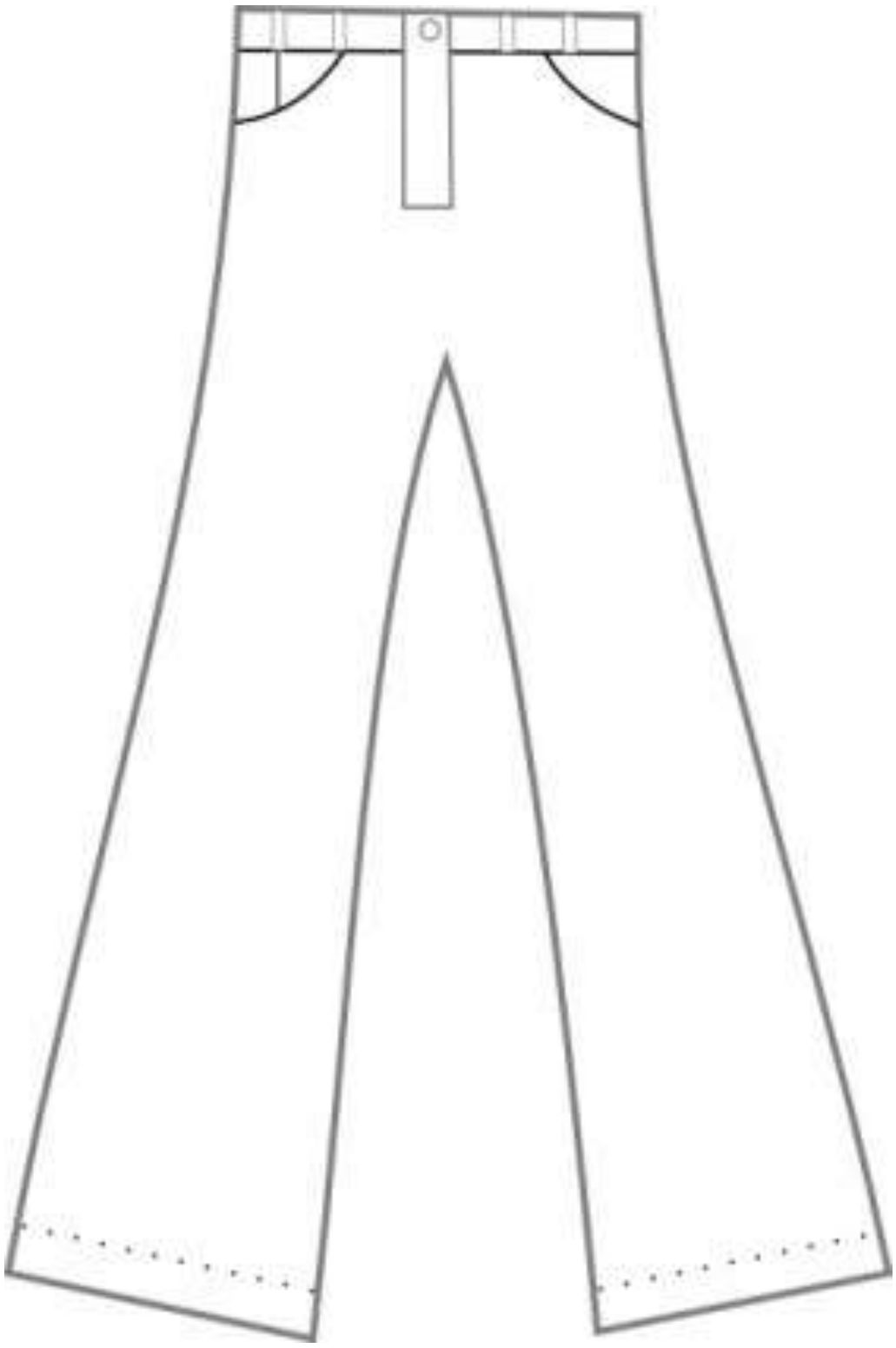
Appendices

Philadelphia Standards

M5.E.3.1.2: Determine the probability of an outcome (e.g., a coin toss, a roll of a number cube) and express as a fraction without reduction.

M6.D.1.2.1: Determine a rule based on a pattern or illustrate a pattern based on a given rule (displayed on a table, chart or graph; pattern must show at least 3 repetitions).

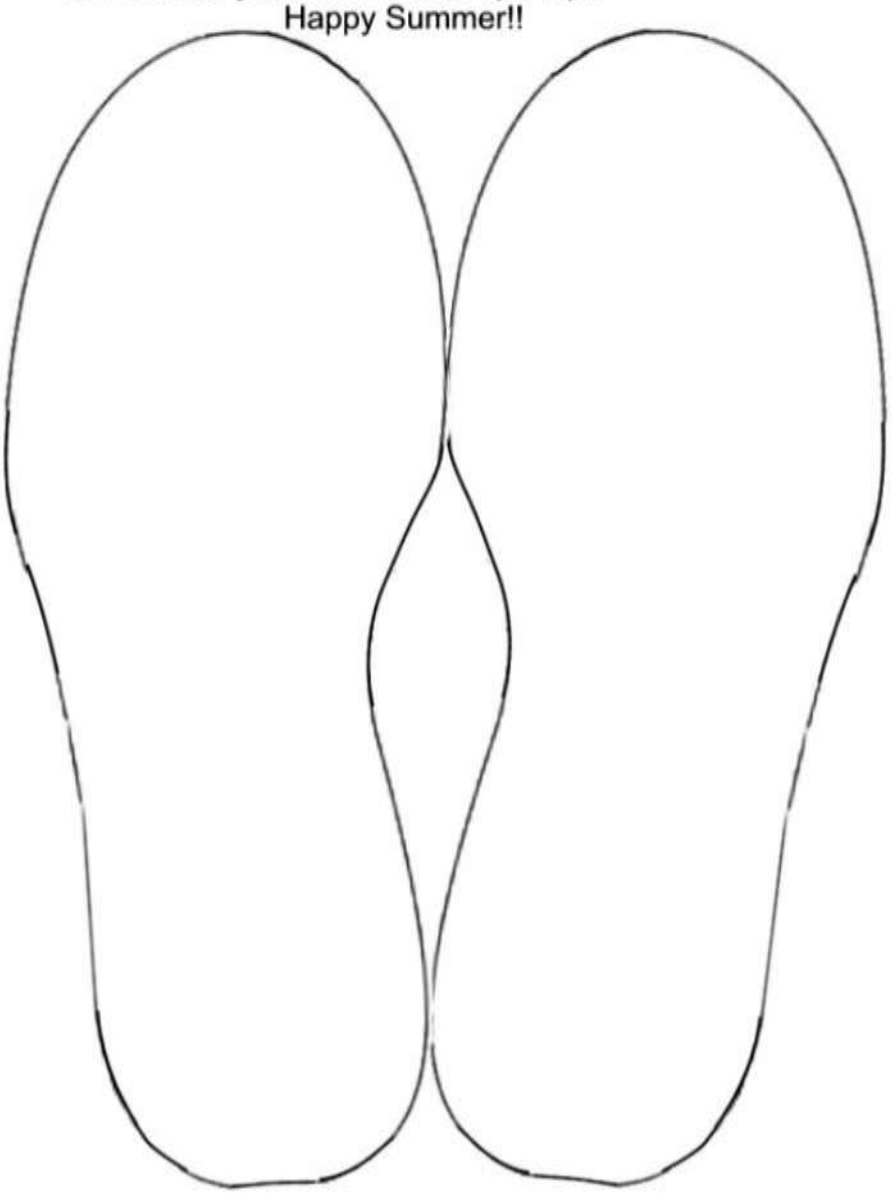
M6.E.3.1.2: Determine/show all possible combinations involving no more than 20 total arrangements (e.g. tree diagram, table, grid).

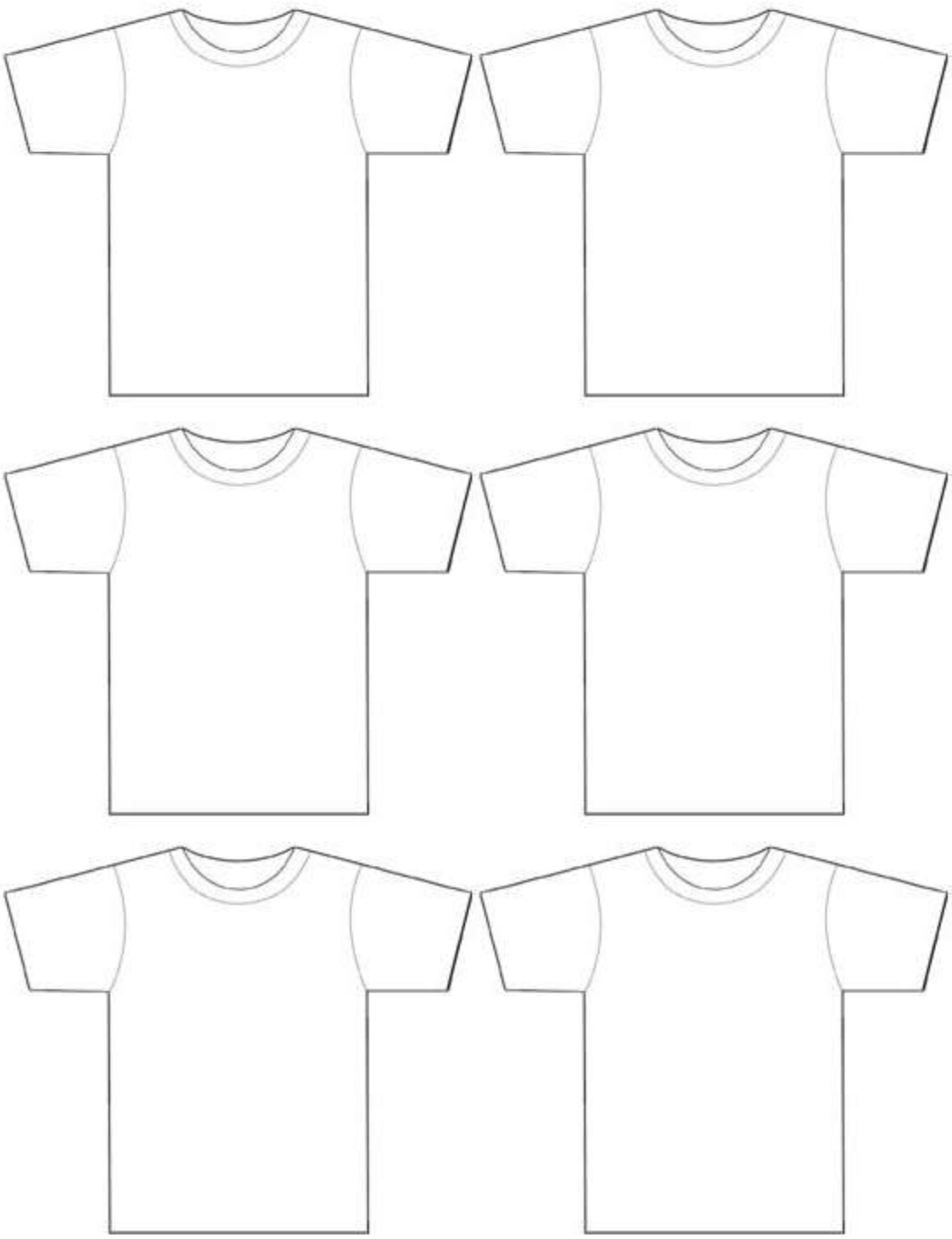


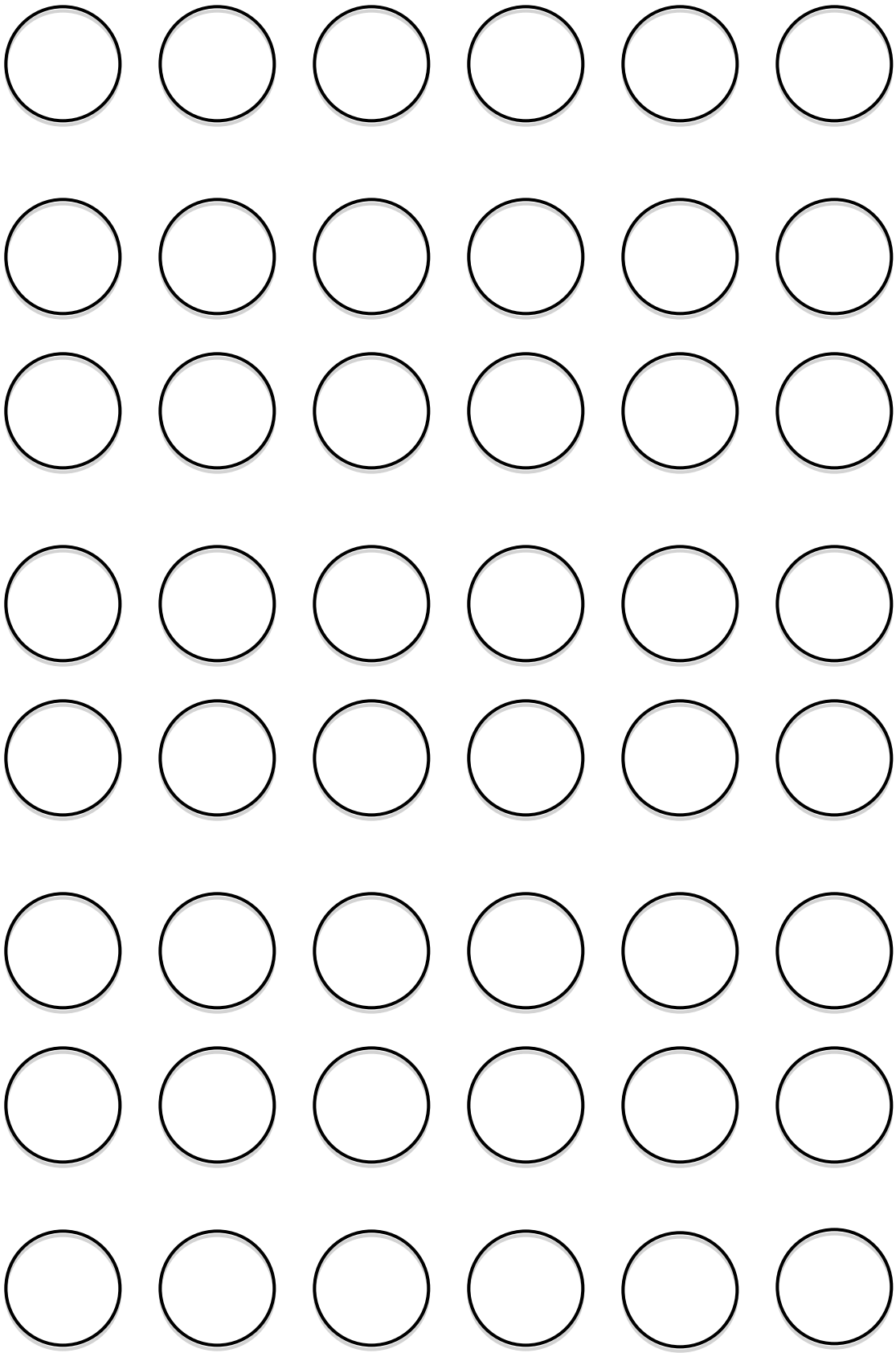
Flip Flop Card Template

1. When cutting out the template make sure to place the attached area on the fold of the Card Stock.
2. Once your template is cut out you can mount the back of the flip flop on Fun Foam for a "realistic" touch.
3. Decorate your card like a Flip Flop!!

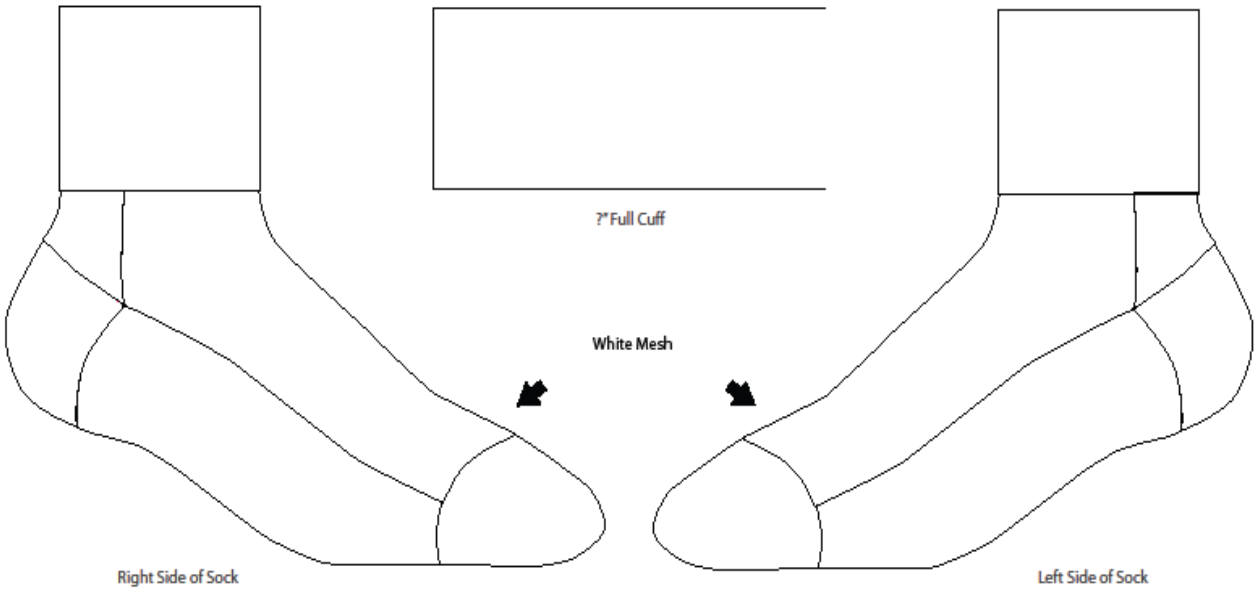
Happy Summer!!







SAVE OUR SOLES



Colors

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