What Are My Chances?
Exploring Probability with Virtual Tools

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Overview

To be literate in the 21st Century requires more than reading, writing and basic math. Handling the amount of information that we encounter each day necessitates going beyond the 3 R's. Our students need to be prepared for accessing, processing and communicating information. To help them understand the meaning of the calculations they perform in acquiring math skills, we need to add probability and statistical thinking to the formula for being literate. Being mathematically literate, or having a sense of "numeracy," requires a precise understanding of mathematical terms, experiences of data based-decision making, and ultimately, a realization that there are many uncertainties in our complex world.

I am the computer teacher at Overbrook Elementary School. My assignments include supporting the curriculum with technology and working with students on projects to enhance their technological skills. This ten-day curriculum unit is designed for fourth and fifth grade students designed to enhance the portion of the School District of Philadelphia's math curriculum dealing with probability. The lesson plans can be used as a two-week unit or spread out over a longer period of time. The activities include opportunities for students to deepen their understanding of basic probability concepts presented in their classrooms with the added benefit of the tools of technology. This unit focuses on using interactive tools and online resources to demonstrate concepts, develop precise vocabulary, play games of risk, design a game, evaluate their understanding, and create a cumulating project.

Rationale

Thinking mathematically requires more than quick calculations. Rather than merely memorizing number facts and repeating methods to solve problems on worksheets, today's math curriculum necessitates helping students gain a deeper understanding of concepts with related applications and examples from everyday life. In his book, *Innumeracy,* John Allen Paulos points out the life-long consequences resulting from the

inability to deal with very large numbers or the probabilities associated with them. From the many examples in his book, Paulos demonstrates that probability, like logic, is not just for mathematicians. It permeates much of our life, but takes time to develop.[i]

So what is probability? We hear it used in our day-to-day lives with words and phrases like: "What are the odds?" "What chance do I have?" Is something 'likely,' 'possible,' 'probably,' or 'unlikely' to happen? Probability is the measure of the likelihood that something will happen. It ranges from serious situations (chance of getting cancer, being involved in an accident) and daily circumstances (weather, sports), to games of entertainment and risk (Lottery, horse racing, cards and dice). It is beneficial for students to learn the vocabulary associated with probability and to remember that, although you can forecast chances in the long run, you can never be absolutely certain of what will actually happen next.

In this unit students will gain familiarity with the vocabulary and the measurement of probability. It is important for them to know that there are several ways of expressing the same meaning. For example, the chance that the top card on a deck of shuffled cards is a Heart is one-fourth. The odds of a Heart being on top are three to one against. A non-Heart is three times as likely as a Heart to be on top. It is easiest to calculate probabilities as fractions when there are equally likely outcomes. In comparing probabilities, percentages (0%-100%), or fractions with common denominators are useful.[ii]

A Brief History

Concepts of probability existed for thousands of years, but probability theory, as a branch of mathematics, didn't emerge until the mid-seventeenth century when gamblers wished to increase their chances for success. Gamblers were crafty enough to figure simple laws of probability by witnessing the events at first hand. In about 1654, Blaise Pascal started to investigate the chances of getting different values for rolls of dice, and his discussions with Pierre de Fermat are usually considered the beginning of the study of probability.

The Dutch scientist Christian Huygens learned of this exchange and soon after published the first book on probability dealing with problems associated with gambling (1657). Because of the great interest in games of chance, probability theory became popular and the subject developed rapidly during the 18th century.

In 1812, Pierre de Laplace introduced a host of new ideas and mathematical techniques in his book, *Analytical Theory of Probability*. Laplace applied probabilistic ideas to scientific and practical problems, not merely games of chance. Since then many others have contributed to the theory of probability and its varied applications.[iii]

Today, probability (and its offshoot, statistics) is used in all types of human activity: assessing risks in medical treatment, forecasting weather, setting discount prices, determining insurance rates, sports outcomes and many areas of the business world. Oddly enough, it has taken us over two hundred years to appreciate the words of Laplace:

"The most important questions in life are, for the most part, really only problems of probability" (*Analytical Theory of Probability*).[iv]

Distinction between Probability and Statistics

Since we frequently hear the words probability and statistics together, it will be important to help students understand the distinctions between these two branches of mathematics. In a nutshell, statistics is all about data and probability is all about chance.

Statistics is a branch of mathematics dealing with collection, analysis and interpretation of data. It deals with data that may or may not be useful for finding probability. Statistics collects data about large numbers of people (or other objects), and uses this data for studying other large groups of people. Statistical data can also be useful by itself, without any connection to probability. It tries to make sense of observations in the real world. For example, census data is used for decision making at all levels of government, and by businesses to determine new growth.[v]

Probability is a mathematical study of chance events that are either random or uncertain. It deals with predicting the likelihood of a future event based on past occurrences. When you need to repeat the same exact experiment over and over to find the probability, you collect data. The only time the data can be used for probability is when all the experiments are the same. You can use data (statistics) from rolling dice one hundred times to find the chances (probability) of rolling a certain number on your next roll. You can't use data (statistics) from studying human activity of a hundred, thousand or a million people to find the chances (probability) of your having a certain disease or the chances of your being involved in an airplane crash on any given day.

Theoretical and Experimental Probability

In many classrooms in the early elementary grades, students are taught to find probability of certain events, such as rolling a "6" on a die, landing on the red section of a spinner, or pulling an ace from a deck of cards. These activities lay some groundwork but as students move into upper elementary and middle years, they can begin understanding the difference between theoretical and experimental probability. Concrete examples and computer-run simulations help students get a clearer understanding of probability.

Probability does not give us ability to predict the future but it helps us realize how likely it is that something will or will not happen. Probabilities are written as numbers between zero and one. A probability of zero means that an event is impossible. A probability of 1 or 100% probability means that the event is certain. In flipping a coin, you have a zero probability of getting heads and tails. Impossible! You have a 100% chance of getting either a Head or a Tail. Certain! The probability of getting Heads or Tails is 50%. Fair!

Theoretical probability is a ratio that compares the number of favorable outcomes to the total number of possible outcomes. For example, the probability of landing a Head while

flipping a fair coin is one chance out of 2. P(H) = 1/2 or 50%. Likewise, there is a 50% chance of landing a Tail. P(T) = 1/2 or 50%.

Experimental probability is determined by the data collected while performing experiments. It is expressed as the ratio of the number of times the event occurs to the total number of times the event is done (trials). It is based on direct observations or experiences. For example, if you flip a coin 10 times, you might get Heads 6 times – 6/10 or 60%. This is higher than the expected (theoretical) probability. If you keep increasing the number of trials, you will notice that the experimental result will get closer to the theoretical probability. Java Script simulations make this an easy task to accomplish. By using online virtual dice, I increased my trials by 100 and recorded the following data:

Number of Trials	100	200	300	400	500
Experimental Probability	47.86%	48.62%	50.08%	50.05%	50.03%

The experimental probability of getting Heads was equal to 50.03%, which is very close to the theoretical probability of 50%.

The Counting Principal

The Counting Principle is used in probability to determine the total number of possibilities when 2 or more coins (or other objects) are involved. For example, if we toss two coins and we want to know what are all the possibilities of getting Heads, we could list them as: HH, TT, HT, and TH: three favorable outcomes out of four possibilities. The probability of getting at least one Head is 3/4or 75%. The probability of getting two Heads would be one out of 4 or 25%.

This counting method works well with a small number of coins. However, for larger numbers, we need a more mathematical approach. One such method is Pascal's Triangle. Pascal's Triangle helps us find the probability of multiple events when there are only two outcomes (Heads or Tails, Male or Female). Each side of the table represents one of two choices, and the numbers from left to right indicates the number of true outcomes for those choices. The total of the numbers added across one row gives the sample space for that experiment.[vi]

This pattern is named after the French mathematician Blaise Pascal (1623-62). By arranging numbers in staggered rows, the Triangle is formed so that each number in the triangle is the sum of the pair of numbers directly above it. To talk about an entry in Pascal's Triangle, we usually give a row number (n) and a place in that row (m), beginning with row 0 and place 0. With this pattern, we can quickly find the probability of tossing m heads when tossing n coins.

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1

For example, the number 15 appears in row 6, place 3. There are 64 outcomes when tossing 6 coins and there are 20 ways to get 3 Heads. The probability is 20/64 or about 31%. Using online tools to generate "n" rows of Pascal's Triangle, we can get a clearer understanding of how the desired "n" outcomes will increase or diminish.

In this unit, students in the elementary grades will be introduced to topics dealing with probability and revisit them at deeper levels of complexity. The activities in this unit will support the math curriculum in fourth and fifth grade classes, and extend interest and understanding through the use of technology.

Objectives

This unit is designed to meet the following objectives:

- to acquire an understanding of probability through digital activities
- to develop understanding of vocabulary associated with probability
- to observe the relationship between theoretical probability and experimental results
- to express probabilities as fractions and/or decimals
- to introduce students to new technologies for experimenting with probability
- to apply strategies to games of chance
- to design and evaluate games of chance
- to create a final project to demonstrate understanding

The Pennsylvania State Standards that address these objectives will be listed in the Appendix.

Strategies

Virtual Manipulatives

Manipulatives allow students to visually examine and explore possibilities while developing concepts. In most classrooms in my school, students are introduced to the concept of probability using manipulatives, such as coins, cards, dice, and spinners to perform trials experiments. These traditional hands-on activities start with flipping a coin or tossing coins or dice, constructing a diagram to record the data, and finally analyzing their individual and collective results. These methods are concrete yet time-consuming and often limit the extent of trials needed to better understand probability.

Virtual manipulatives or interactive, web-based programs allow students to construct knowledge about probability using dynamic objects (coins, cards, dice, and spinners) in repeated trials of an experiment. With a resultant computerized display of data, students can immediately analyze the visual data, and explore other 'what if's?' to further their thinking and deepen their understanding. Virtual manipulatives can lead to more complex, richer understandings of concepts.

Smart Board and Java-Based Tools

The Smart Board contains computer-based mathematical manipulatives for designing interactive lessons and activities. The online Java-based mathematical tools and editors allow exciting new approaches to mathematical instruction. With these tools and web-based simulations, students are actively engaged, discover relationships and pose new questions. We will be using Spinner, Dice, and Coins for the activities in this unit as well as interactive tools for vocabulary development and review.

Visual Representations

Visual representations allow students multiple ways to display and interpret information such as tree diagrams, arrays and lists. When using computer simulations for mathematical exploration and experimentation, students benefit from visualizing results. When explaining their models, they are putting their thinking into words. This may lead to new questions and discussions, which will in turn promote deeper thinking and better understanding.[vii]

Games

In playing games of risk, students will discover the difference between choice and chance. They will learn that even games that have choices can be mostly chance for a player who makes choices without any strategy. When choice is embedded in the game, students can use strategies based on probability to make decisions. As they replay games, they will have opportunities to try out and discuss strategies for improving their chances of success. The Game of Pig[viii] and the Game of Skunk[ix] are two games used in this unit that will help students made decisions while playing, develop strategies, and evaluate manner of playing to improve rate of success.

Classroom Activities/Lesson Plans

The following lesson plans will be used in the computer room to supplement the fourth and fifth grade curriculum's probability strand. All students will have access to an Internet-connected computer. The Smart Board in the room will provide introductory and summative materials.

Lesson 1: Introduction: Taking Chances

Students will have had some exposure to the topic of probability in their classrooms. I will use the Smart Board in the computer Lab to review vital vocabulary and provide examples.

Objective: To review definitions related to chance and match them with appropriate percentages.

Duration: One period- 45 minutes

Procedure: Using the anagram tool on the Smart Board, have one or two students come up to figure out the word CHANCE. Provide hints along the way if needed. Ask a few students to explain their meaning of the word *chance*, and then ask them to refer to a chart with chances varying from 0% to 100%. They will then fill in the chart with words used to describe *chance:* certain, impossible, likely, unlikely, equally likely, very likely.

Percent Chance	Meaning
100	Certain
90	Very Likely
80	
70	More likely
60	
50	Equally likely
40	
30	Less likely
20	
10	Unlikely
0	Impossible

After the chart is filled in and we have discussed its meaning, I will ask students to convert the percentages to fractions to express chances out of 10. Students will then use the software *Kidspiration* to depict scenarios with the given words. For example, to illustrate 0%, a student will explain the meaning of impossible (it can't happen) and include examples. I will encourage them to give one from reality (dogs flying), and one related to the topic of chance, such as winning the lottery without buying a ticket.

Assessment: Students will construct a timeline and plot the words associated with chances: impossible (0), unlikely, even chance, likely and certain (1). They will have the option of annotating with small graphics, such as coin, dice, marble jar, and deck of cards. This will prepare them for understanding probability as a number between 0 and 1.

Extension: Create a table similar to the one used above and record weather predictions and results for 7 days. How accurate were they? Collect data from various meteorologists and decide which one was most accurate.

Lesson 2: What are my Chances?

Objective: To use virtual manipulatives to explore choices and chances in games of probability.

Duration: One period- 45 minutes

Procedure: I will begin by using the dynamic die on the Smart Board Tools. I will ask a student to select a number from 1 to 6 and then roll the die. After several students have had a turn we will talk about the probability of rolling a number from 1 to 6 on the die. I will ask them: What are the total outcomes for rolling the die and how many ways can their selected number show up? With input from the students, we will determine that the probability is the number of possible outcomes/total number of outcomes. Then using the coin and the spinner, we will continue to calculate probability of Heads or Tails on the coin, or a specific color on the spinner. Students will take turns recording the data from these activities

Students will then use virtual manipulatives to further explore the probability of favorable outcomes. At the virtual die server (http://dicelog.com/dice),[x] students will first explore the possibilities and outcomes of rolling one, two or three dice. Then they will conduct an experiment by setting the Number of Dice to Roll (2) and Number of Rolls (10). After they click "roll," the dice generator will list a picture of each die along with a list of sums. Students can then increase the number of rolls as time allows and notice which combinations come up more frequently.

Assessment: Using Microsoft Word, students will insert a table into a document and list the ways each number from 2 to 12 can be made rolling two dice.

2	3	4	5	6	7	8	9	10	11	12
1+1	1+2	1+3	1+4	1+5	1+6	2+6	3+6	4+6	5+6	6+6
	2+1	2+2	2+3	2+4	2+5	3+5	4+5	5+5	6+5	
		3+1	3+2	3+3	3+4	4+4	5+4	6+4		
			4+1	4+2	4+3	5+3	6+3			
				5+1	5+2	6+2				
					6+1					

Then they will answer the following questions: What is the total number of possible outcomes? How many numbers can only be made in one way? Two ways? Three ways? Four ways? Five ways? Six Ways?

What is the probability of rolling two dice and getting a sum of 7? (Students should reason that there are 6 ways to roll a combination of 7. So the probability would be 6/36 or 1/6, or about a 17% chance.)

I will provide more examples if needed: A sum of 11? A sum of 6? Etc.

Extension: Nines and Tens
Explain why it is that when you throw two dice you are more likely to get a score of 9 than of 10.

Lesson 3: Theoretical and Experimental Probability

Objective: To discover the relationship between theoretical and experimental probability

Duration: One period- 45 minutes each

Procedure: Before beginning the activities, I will display the words "experimental probability" and "theoretical probability" on the Smart Board and explain that we will be working on activities to discover the difference between them.

Using the coin toss on the Smart Board, I will ask the students what are the possible outcomes of tossing the coin? (Heads or Tails) What is the probability of getting Heads? Tails? We will define this expected outcome as the ***theoretical probability***: number of favorable outcomes/number of possible outcomes. We are finding the probability of an event with equally likely outcomes. There is 50% chance of getting Heads and 50 % chance of getting Tails.

If we toss a coin 10 times, how many Heads should we expect to get? (5)
Have a student use the Smart Board to toss a coin 10 times with another student recording results. How many Heads did we get? We will define this actual number of outcomes with Heads as the ***experimental probability***: number of times the event occurs/the total number of times the event is done (trials).

Next students will use the virtual coin to explore differences between theoretical and experimental probability. Students will be directed to a simulated coin toss website[xi] http://syzygy.virtualave.net/webwork/javascript/cointoss.htm and repeat the coin toss in increments of 100. They will record results on a worksheet and convert results to percents.

Number of Trials	100	200	300	400	500	600	700	800	900	1000
Expected Heads/Tails										
Actual Heads										
Percent Heads										
Actual Tails										
Percent Tails										

When they are finished recording their results, we will discuss the following:
What was the theoretical probability of getting Heads or Tails when tossing a coin 100 times, 200 times, etc.?
Did you get the number of Heads or Tails that you expected when tossing the coin?
What do you notice about the percent of heads or tails as you increased the numbers of trials? Is it getting closer to 50%?

Assessment: Students will answer these questions in their own words
Explain the difference between theoretical and experimental probability.
If the theoretical probability of tossing Heads or Tails is 50%, does that mean if you toss a coin twice you will get 1 head and 1 Tail?
What does it mean when we say that your chances of getting Heads or Tails (outcomes) are equally likely when tossing a coin?
What other experiment could we conduct to show the difference between theoretical and experimental probability (tossing 2 coins, using spinners, dice, etc.)?

Extension: At Least One. Use a tree diagram to determine if you will get at least one Head when tossing a coin 3 times. Does this data help you find the probability?[xii]

Lesson 4 and 5: Game of Risk

Day 1: The Game of Pig

Duration: One period- 45 minutes

Objective: To play The Game of Pig (one die) to better understand "choice versus chance" and how it applies to mathematical probability.

Procedure: I will begin by using one die on the Smart board. I will ask students to figure out what is the probability of rolling a 1? What is probability of rolling a number other than 1? We will fill out a table to determine probability.

	# Favorable outcomes	# Possible outcomes	Probability
For a 1	1	6	1/6 or 16 %
For a 2,3,4,5,6	5	6	5/6 or 83 %

After experimenting with this, I will explain to the students that we will use the die to play a game called "Pig." In this game of risk, they have to decide to roll or hold the dice.

The game of Pig is a dice game in which two players race to reach 100 points. For each turn, a player repeatedly rolls a die until either a 1 is rolled or the player holds and scores the sum of the rolls. The first player to score 100 or more points wins the game of Pig.

If the player rolls a 1 (called a pig), that player's turn ends, no points are earned and the other player takes a turn.
If the player rolls a 2, 3, 4, 5 or 6, the turn continues and the player scores the sum of the rolls (i.e., the *turn total*).
When a player chooses to hold, he or she stops rolling and all the points rolled during that turn are added to his or her score.

I will ask for two volunteers who will demonstrate the game.

Next students will use an online tool to play the game against the computer.[xiii] (http://cs.gettysburg.edu/projects/pig/)They will click *roll* or *hold*. The key decision facing a player is how large a turn total should be risked to possibly get an even larger total.

Assessment: Students will discuss strategies used when playing game. What motivated them to hold or roll? How often did a "1" turn up? Did you play it safe? Did you take a risk?

Day 2: The Game of Skunk

Objective: To play The Game of Skunk (pair of dice) to better understand "choice versus chance" and how it applies to mathematical probability.

Duration: One period- 45 minutes

Procedure: Skunk can be played with the whole class. Our first game will be for practice. To prepare for the game students will create a scorecard on their individual computers.

S	K	U	N	K

Before playing the game, I will ask students to recall from our previous lesson that there were 36 possible outcomes when rolling a pair of dice. I will display this chart and explain the rules for playing SKUNK:

There are 5 rounds in the game corresponding to the letters of SKUNK.
The object of the game is to get the highest score possible through the 5 rounds.
All students will stand up to start the game. I will roll the dice on the Smart Board. The students have a choice before each roll whether to remain standing or whether to sit down.
If you are standing and the number 1is NOT rolled, YOU record the points on the S of the core card
If you decide to sit down during the round, you total your points and wait for next round.

If you are standing when a “1” is rolled, you lose ALL your points for that round.
If you are standing when “double 1’s ” are rolled, you lose ALL your points for all rounds up to that point.
If a "1" doesn't occur, you may choose either to try for more points on the next roll (by continuing to stand) or to stop and keep what he or she has accumulated (by sitting down).
The game continues until all rounds are played. Then you total your points for a final score. The highest score wins.

Since rolling the number “1” will result in no accumulations of points, I will ask students to refer to the chart and determine how many possible outcomes will allow them to score points. (There are 25 possible outcomes for scoring points in the game.)

Assessment: After we play one round for practice, I will ask students if there are any strategies that might help them achieve a higher score. What part is chance? What part is choice? For the next game of SKUNK, I will ask students to see if they can improve their score. Upon completion, we will discuss strategies and rules of probability. About how many good rolls occurred before a “1” was rolled? Does this help you develop a strategy?

Lesson 6: Show What You Know

Objective: To explore online resources for learning about probability.

Duration: One Period- 45 minutes

Procedure: Students will be introduced to the Probability Section of the Skillswise website.[xiv] The site offers them games, fact sheets, worksheets, quizzes, and suggestions for exploring and investigating concepts of probability.

Students will have time to complete at least one quiz in class and print out their results that provide feedback and explanations. They will be encouraged to continue to visit the site for review and test preparation for PSSA.

Lesson 7 and 8: Game Design

Objective: Students will apply the knowledge and experience from previous lessons to design a game of risk.

Duration: Two Periods- 45 minutes each

Procedure: I will explain to students they will work with a partner or with a small group to design their own game of risk. They may choose to take the basics of PIG and SKUNK and modify the rules to reflect greater or lesser risk. They many use dice, cards, spinners

or coins for their games. I will encourage them to use the tools we have on the Smart Board as well as online tools.

We will explore some interactive online games as models. Students will discuss the fairness of games and the risk involved.

Racing Game: A red and blue car race to finish line depending on roll of die: Even – red moves; odd-blue moves. Players can change what numbers determine the car moves and the number of segments to finish line.
http://www.shodor.org/interactivate/activities/RacingGameWithOneDie/#printing

Coin Toss: Players can determine how many tosses and how data is displayed.
http://www.shodor.org/interactivate/activities/Coin/

Guidelines for Game:

- work in pairs or in small groups to create a game involving choice and chance
- give a short description of the game
- include clear instructions for playing your game
- try the game out with another class member for feedback
- include a reflection piece: how does this relate to probability that we explored these past two weeks? what were the choices and chances involved? was the game fair or unfair? how can the game be improved?
- present game to the rest of the class

Assessment: Students will select the top 5 games, explaining their choices. These will be compiled in a game folder and made available to other classes in the school.

Lesson 9 and 10: Cumulative Project

Objective: Students will use a variety of formats to demonstrate their understanding of the content and skills covered in the implementation of this unit.

Duration: Two Periods- 45 minutes class periods; independent time and extra support as needed

Procedure:
We will begin by brainstorming a list of possible projects and the tools available:

- use Flip Camera to video students playing a game and voice over with a commentary about taking chances and risk involved
- using PhotoBooth, produce 30-second clips explaining vocabulary related to probability
- create a Comic Life (comic book format) to give some key points in the history of probability

- record a Podcast using GarageBand: interview other students about probability in everyday life (weather, sports, games, etc)
- prepare a PowerPoint Presentation about careers involved with probability
- create a Blog (Kidblog.org) to post topics and questions about probability
- prepare a Smart Board presentation to explain and illustrate a concept or play a newly designed game
- create a poster to introduce probability to a lower grade in the school
- create an Annotated Bibliography of books dealing with probability; include fiction and non-fiction; organize according to grade levels for our school (K-5)
- write (illustrate) your own book about probability

I will provide students with a review of the tools available. They will be familiar with most of them. Students may choice to work independently, with a partner, or in a small group. Once they have decided on the format, I will give mini lessons for tips on using the tools selected and they will sign up to attend.

Guidelines for Presentation:

- stick to presentation time duration: between 3-5 minutes
- complete storyboards prior to multimedia creation
- adhere to Copyright and Fair Use Guidelines (posted in classroom)
- sign up for equipment (cameras, laptops, microphones)
- ask for help- don't wait until last minute
- rehearse your presentation

Assessment:
Students will be assessed on the following points:

- Preliminary work- create storyboard or outline
- Design- demonstrate proficiency with format selected; exhibit creativity
- Content- include information that is relevant and interesting
- Presentation- express thoughts and convey information accurately

Annotated Bibliography/Resources

For Teachers:

Access to the Core Curriculum: Strategies Guide. School District of Philadelphia, 2006.

Apostol, Thomas: "A Short History of Probability," in *Calculus, Volume II* Wiley and Sons, 1969. <http://www.cc.gatech.edu/classes/cs6751_97_winter/Topics/stat-meas/probHist.html>

Burns, Marilyn. *Math By All Means: Probability, Grades 3-4*.Math Solutions, 1995. Burns emphasizes critical thinking through careful exploration of probability. There are whole class, small group and individual lessons and explorations.

Haigh, John. *Taking Chances: Winning with Probability*. Oxford Press: New York, 2003.
This book provides an entertaining look at various ways of understanding chances and risk. Using examples from everyday live, such as sports, weather, lottery, Haigh examines common pitfalls resulting from misunderstanding probability.

Healy, Lulu, Hoyles. "Visual and Symbolic Reasons in Math: Making Connections with Computers." Thinking and Learning, 1999.
This is an excellent review of the importance and necessity of using visual representation during math instruction and exploration.

Paulos, John Allen. *Innumeracy: Mathematical Illiteracy and Its Consequences*. Hill and Wang: New York 1988
In this book, Paulos examines our everyday life and provides practical advice on how to improve both education for our young students and strategies for those out in the world.

Holmes, Susan. *Probability by Surprise*. Stanford University: Statistic Department, Sequoia Hall http://www-stat.stanford.edu/~susan/surprise
This resource was chosen as by the National Council of Teachers of Mathematics Illuminations Project. Of particular interest is the 'probability tree' generator.

For Students:

Gillam, MS. *Culminating Project Ideas*.
<http://web.me.com/cmstechnology/CMSTechnology/8thGrade.html>
Although this is geared for 8th grade, I found it useful for elementary students.

Johnnie's Math Page: http://jmathpage.com/JIMSProbabilitypage.html
A guide to the interactive math tools and activities for kids: spinners, dice, coins and much more

Knapp, Brian. *Math Matters; Chance and Average*. Grolier Publishing: Danbery, CT, 1999.
This is a beautifully illustrated book explaining the vocabulary and basic concepts of probability.

NRICH: University of Cambridge, 2007 http://nrich.maths.org/4308
This website offers challenging math problems and solutions, games and articles for students aged 5-19.

Online Assessments:
http://www.bbc.co.uk/skillswise/numbers/handlingdata/probability/factsheet.shtml
http://www.math-play.com/Probability-Game.html
Students will enjoy testing their knowledge of probability in these playful formats providing them instant feedback and explanations

PBSKIDS.ORG: < http://pbskids.org/cyberchase/games/probability/>
On its Cyberchase section, the coin toss game offers kid friendly explanation and visual representation of accumulated data

Videos: **http://www.math-videos-online.com/probability-videos.html** Here is a collection of free online videos on different topics dealing with probability. Some probability videos are followed by online quizzes and interactive games designed to reinforce the concepts taught in each movie.

Appendix/Standards

Resources

Computer lab with Internet access for each student
Flip Camera
Digital Camera
Smart Board or other IWB
Software
Microsoft Word, Excel, and PowerPoint
Kidspiration or Inspiration
iPhoto and iMovie
PhotoBooth
GarageBand
Comic Life

Pennsylvania Academic Standards for Mathematics

Students will have a variety of experiences in the area of Probability and Predictions.

2.7.5 (A): Predict and calculate the likelihood of simple events.
2.7.5 (B): Predict and determine why some outcomes of a particular event are certain, more likely, less likely, equally likely, or impossible.
2.7.5 (C): Express probabilities as fractions and/or decimals.
2.7.5 (D): List all possible combinations and arrangements of outcomes of an experiment): (e.g., tree diagrams, matrices)
2.7.5 (E): Compare predictions based on theoretical probability and experimental results.

Pennsylvania Academic Standards for Science and Technology

Students will use a variety of technological and information resources to gather and synthesize information, and to create and communicate knowledge.

3.6 (B): Technology Education: Demonstrate the ability to communicate an idea by applying basic sketching and drawing techniques.

3.7.4 (C, D, E): Technological Devices: Identify and use simple graphic and presentation materials generated by the computer.
3.8 (A, C): Science, Technology and Human Endeavors: Apply the technological design process to solve a simple problem

Pennsylvania Academic Standards for Reading, Writing, Speaking, and Listening

Students will use a wide range of strategies to acquire the knowledge and skills needed to:
1.2.5 (E): Read, understand, and respond to essential content of text and documents in all academic areas
1.6.5 (A): Listen critically and respond to others in small and large group situations. Respond with grade level appropriate questions, ideas, information, or opinions
1.8.5 (C): Produce an organized final product that presents the results of research findings, using appropriate visual aids and citing sources.
1.9.5 (A): Use media and technology resources for problem solving, self- directed learning, and extended learning activities.

Footnotes

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i. Paulos, John Allen. Innumeracy: Mathematical Illiteracy and Its Consequences. Pp 3,7
ii. Haigh, John. Taking Chances pp. 4-5.
iii. Apostol, Thomas: http://www.cc.gatech.edu/classes/cs6751_97_winter/Topics/stat-meas/probHist.html
iv. LaPlace, Pierre-Simon. Analytic Theory of Probability. (1812). Retrieved from Encyclopedia Britannic, 2011 <http://www.britannica.com/EBchecked/topic/591039/Analytic-Theory-of-Probability>
v. Skiena, Steven. "Probability versus Statistics." Retrieved from <http://www.cs.sunysb.edu/~skiena/jaialai/excerpts/node12.html>
vi. Math forum. < http://mathforum.org/dr.math/faq/faq.pascal.triangle.html>
vii. http://www.netc.org/focus/strategies/nonl.php
viii. Pig is a folk jeopardy dice game described by John Scarne in 1945, and was an ancestor of the modern game Pass the Pigs (originally called Pig Mania).
ix. NCTM. Illuminations. "The Game of Skunk." <http://illuminations.nctm.org/LessonDetail.aspx?id=L248>
x. http://dicelog.com/dice),
xi. Jeff LeMieux, 2002. <http://syzygy.virtualave.net/webwork/javascript/cointoss.htm>
xii. These and other challenges can be found at NRICH: < dhttp://nrich.maths.org/7286 >
xiii. http://cs.gettysburg.edu/projects/pig/piggame.html
xiv. http://www.bbc.co.uk/skillswise/numbers/handlingdata/probability/factsheet.shtml