The Importance of Surface to Volume Ratio (SVR) in Everyday Life.

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Overview:

In this curriculum, students will explore the role of surface area of different plane figures (laminas) i.e. rectangles, squares, trapezoids, rhombuses, parallelograms, circular discs, annular discs etc. Surface areas of solid figures (i.e. 3-dimensional figures) are equally important and it is related with volume. Special emphasis is given to the exercises involving surface area and volumes of prisms, pyramids, cubes, cones, spheres and hemispheres.

Surface areas of different figures play a very important role in daily life. Estimating the cost of ploughing a field, the number of trees to be planted in a field (agriculture); the number of tiles to be laid on the floor or ceiling of a building, cost of painting the interior and exterior of a building (construction or architecture), involves the calculation of surface area and cost per square unit. Even the big dome of planetarium, where we can see the vastness of sky, different planets, stars and constellations (astronomy) involves the surface area for its construction.

Surface area also involves in the phenomenon like evaporation and chemical reaction. As in a chemical reaction, if the surface area of chemicals is increased, the rate of chemical reaction is also increased. More the surface area exposed the faster is the rate of evaporation. Students may observe if sugar powder is added in coffee, the reaction is quicker than if sugar crystals of the same quantity are added.

In present day laboratories, chemists, metallurgists and material scientists routinely design and produce new materials to meet specific needs. Solid state physicists explore solids such as semiconductors and tailor them to meet the needs of an information society.

Terms, such as, surface tension of a liquid can also be studied by knowing the surface area of a solution. Pressure inside or outside water bubble depends on the surface tension of spherical bubble, which in turn, depends on the surface area. Surface properties of different objects can be well studied by using technology, whether it is a small and simple calculator or large and complex microscopes used in the field of nanotechnology.

Rationale:

Definitions:

Definitions used frequently in my curriculum are:

1. Density
2. Surface
3. Volume
4. Elastic
5. Inelastic
6. Scaling
7. Surface tension
8. Evaporation

1. Density: It is the most important property of solids, liquids, and gases and can also be referred as a measure of how tightly the material is packed together. It can also be defined as: how much matter (mass) is squeezed into a given space (volume). Mathematically, it is the ratio of mass and volume or it is the amount of mass per unit volume d= m/v. Unit of mass is kilogram and unit of volume is meter cubed so unit is density is kg/m . Other units of density are g/cm or g/ml. When an object is broken into pieces, the physical property such as, density does not change, it remains the same. The density of crown made up of pure gold is the same as that of pure gold coin from the mint.

2. Surface: It is the region or area occupied by an object. It is the malleability of an object or metal to the metallurgist or to a goldsmith or to a blacksmith. Gold, the most malleable metal or element can be beaten into a very thin sheet. On the other hand, some metals, when beaten to make sheets, break into pieces showing brittleness. It all depends upon the internal cohesive forces that bind the particles of metals.

3. Volume: Some students get confused with the difference between volume and mass. However, these are entirely different terms. Volume is the measure of space or it is the capacity of a container to hold the liquid. Unit of volume are cubic meters, cubic centimeters, or liters. One liter is equal to thousand cubic centimeters. Equal size bags of cotton and nails may have equal volumes but unequal masses due to the physical property and density of cotton and nails. Volume of regular solids can be determined by using mathematical formulas, some of which my students are already aware of, such as;

Volume of a cube= (side) = s
Volume of a rectangular box= l.w.h.
Volume of a cylinder= π r h where r is the radius of base and h is the height.
Volume of a sphere= 4/3 π r
Volume of a hemisphere= 2/3 π r
Volume of a cone= 1/3 π r h

Volume of the prisms and pyramids can be easily calculated by using formulas.

Volume of a Prism= B × h where b is the area of base of a prism and h is the height of prism.
Volume of a Pyramid= 1/3 × B × h

As a cone is a pyramid, its volume can be found by using above formula. Base of cone is a circle, therefore B= π r . Thus, V = 1/3 π r h

4. Mass: To most people, mass is always confused with weight. Many consider the mass of an object as the weight of object. If a teacher asks a student, what is his/her weight, the answer often is a numeric figure, 50 kg or 120 lbs or so. In the realm of science, students reply contains their mass not their weight. Mass is defined as the quantity of matter contained in an object. Mass is measured in kilograms, pounds or tons. Smaller units of mass are grams, milligrams, micrograms, nano grams. The appropriate unit used depends on the object. For example, mass of a book can be expressed in kilograms but mass of a sand particle can be expressed in very small units like milligrams or micrograms or nano grams. The following conversion units are used to perform calculations in metric system.

I kilogram = 1000 grams
I gram = 1000 milligrams
1 milligram = 1000 microgram

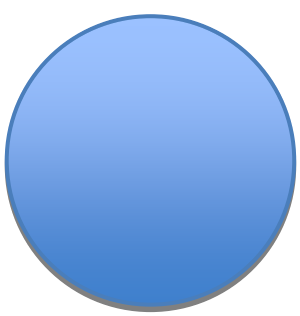
1 microgram = 1000 nanogram

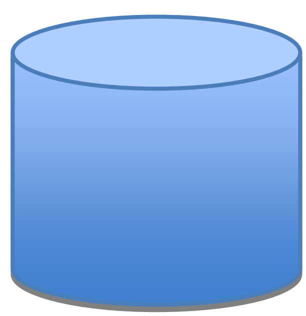
Mass of an object is measured by a physical balance or digital scale. Another way to measure the inertial mass of an object is by exerting a known force on it and measuring its acceleration.

5. Elasticity: Elasticity is the ability of an object to restore its form or to return to its original form. The object which regains its original form quickly is called more elastic. Again, some people are very confused with the concept of elasticity. If you ask them, which of the two, steel or rubber is more elastic? Their instant answer is: rubber. But from the scientific point of view, steel is more elastic than rubber because particles of steel return very quickly to the original form, when deformation forces are removed.

Some of the objects do not return to their original form, even though, the deforming forces are removed, such objects are called plastic. For example: clay and plasticine. Terms elastic and inelastic are also used in terms of kinetic energies. In an elastic collision, the total kinetic energy of the objects is the same before and after the collision. However, if, during a collision some energy is changed into other forms, the collision is called inelastic collision.

6. Surface - tension: We often notice that dew drops on spider webs and falling drops of milk or oil are nearly spherical. Even a drop of water on a smooth surface forms a rounded shape, while a drop of alcohol tends to flatten out. All of these phenomena are examples of surface tension. Surface tension is the tendency of the surface of a liquid to contract to the smallest area. And this is the reason that drops of water and dewdrops are spherical in shape. Amongst all the solid figures for the same volume, sphere has the smallest surface area.





Volume of a Sphere: $4/3\ \pi\ r$ = Volume of a Cylinder: $\pi r .h$

Therefore, $h = 4/3\ r$

Surface area of Sphere= $4\pi r$

Surface area of Cylinder = $2\pi r \; + 2\, \pi r.\ 4/3\ r = 14/3\ \pi r$

Surface area of sphere is less than the surface area of the cylinder.

Surface tension is a result of cohesive forces. Beneath the surface of a liquid, each particle of the liquid is attracted equally in all directions by neighboring particles. As a result, there is no net force on any of the particles beneath the surface. At the surface however, the particles of the liquid are attracted downwards and to the sides. As a result, the surface of water experiences a downward force, pulling the surface downwards and showing a concavity upwards. This is the reason behind water in a container showing lower meniscus or concavity upwards. This phenomenon is also called surface tension of water because its surface acts like a stretched membrane and the water spider or mosquito can stand on water surface easily.

7. Evaporation: The particles in a liquid move at random speeds. Some are moving slowly and others rapidly. The temperature of liquid is dependent on the average kinetic energy of its particles. Suppose a fast moving particle is near the surface of liquid. If it can break through the layer, it will escape from the liquid. This escaping of particles from the surface is called evaporation. More the surface area more is the evaporation. Evaporation always causes cooling due to loss in energy. It is because; each time a particle with higher than average kinetic energy escapes from the liquid, the average kinetic energy of the remaining particles decrease. A decrease in kinetic energy is a decrease in temperature. The result is the cooling effect of evaporation. For example, when we pour few drops of gasoline or alcohol or ether into the palm of a hand, it evaporates easily there by causing a cooling effect.

8. Scaling: The proportions of things in nature are in accord with their size. The study of how size affects the relationship between weight, strength and surface area is known as Scaling. As the size of a thing increases, it grows heavier much faster than it grows stronger. For example, we can support a toothpick horizontally at its ends and we'll notice no sag. But support a tree of the same kind of wood horizontally at its ends and we'll see noticeable sag. The reason behind this is that, the tree is not as strong per unit mass as the toothpick. Weight depends on volume, and the strength comes from the area of cross – section of limbs, tree limbs or animal limbs.

Let us consider an example of a solid cube of matter with its edge equaling one centimeter. The surface area of its base is one squared centimeter and its volume is one cubic centimeter. Surface area of base to volume ratio is 1:1. If linear dimensions of the cube are doubled, the surface area of base would be four squared centimeter, however, the volume would be eight cubic centimeter. Surface area of base to volume ratio is 1:2. If linear dimensions of the cube is tripled, the surface area of base would be nine squared centimeter, however, the volume would be twenty seven cubic centimeters. Surface area of base to volume ratio is 1: 3; similarly quadrupling the linear dimensions , the new ratio would be 1:4.

From this example, we generalize that when linear dimensions are enlarged, the cross sectional area grows as the square of enlargement factor, whereas volume and weight grow as the cube of the enlargement factor. The volume or weight increases much faster than the corresponding cross sectional area.

Surface Area/ Volume Ratio:

From the discussion of terms, volume and surface area, we concluded that weight depends on volume, while strength depends on area of cross section.
However to find surface/volume ratio, let us consider a cube of side "S".

Surface Area= 6S ; Volume = S ; Surface Area/Volume= 6/S

For: S = 1, 0.1, .01, .001 …………

This ratio becomes 6, 60, 600, 6000…………..

On a macro scale, this ratio is 6, 60………….

On a micro scale, this ratio is 6000.

On a nano scale, this ratio is 6,000,000,000.

Thus, this discussion shows that surface area for miniature substances compared to their volume (or weight) is very large. Have you ever noticed that, a clever cook at a restaurant, to peel 10 kg of potatoes, prefers small potatoes or large potatoes? He prefers large potatoes because peeling 10 kg of large potatoes is less than 10 kg small potatoes.

Crushed ice exposes more surface area, so cooling a drink with crushed ice is faster than a single ice cube of the same mass. Rusting of iron or other metals also depends on surface area. More the surface area of iron is exposed, faster it rusts. Iron wool or iron fillings get rusted sooner than single chunk of iron with same mass. Thin French fries cook faster in oil than fat fries. Flat hamburgers cook faster than meatballs of the same mass.

The volume increases much faster than the corresponding enlargement of cross sectional area. This principle applies to any object or shape. The African elephant has less surface area compared to its weight than other animals. It compensates for this with its large ears, which significantly increases its radiating surface area and promotes cooling.

Wep notice that large animals can really harm themselves if they fall from a height with no parachute. Small insects on the other hand, do not harm themselves equally when compared to large animals. An insect can fall from the top of a tree to the ground below with no harm. The surface area to volume ratio is in the insect's favor, in a sense; the insect is its own parachute.

At rest, all warm blooded animals lose the same amount of heat per unit area of skin. Therefore, the amount of food they require is proportional to their total area, not their volume or weight, although the total surface area in relation to the volume or weight is a factor. Because a small animal has a relatively large total area compared to its volume or weight, the animal requires a relatively large amount of food each day.

At Micro Level:

The prefix micro means; one-millionth part of a meter. Before the idea of nanotechnology emerged and advanced, the term micro level was used to describe smallest units of an object or shape. Micro structured plastic devices, micro pumps, peristaltic pumps, polycarbonate sheeting and precision engineering were the prominent fields in which micro technology was used. Micro pumps are further divided into different types, depending on their functions such as; water pumps, DC power pumps, booster pumps, submersible pumps, centrifugal pumps, hydraulic pumps etc. [pump kits for concentrated acids and alkalis, like chromic acids, nitric acid, sulfuric acid, and sodium hydroxide use a unique barrel pump designed in PVDF (polyvinylidenfluoride) throughout the world. Immersion length of the type of pump is 1000 to 1200 mm.

Polycarbonate sheeting also uses technology at micro level. Polycarbonate offers all of the clarity and beauty of acrylic with an impact resistance that is unparalleled in glazing material. Polycarbonate is used in applications such as; machine guards and security windows that require the optical clarity and the assurance of safety.

Polycarbonate is relatively light weight and is fairly rigid. It can be easily sawed, machined, heat formed, cold formed, cemented and painted. They are high molecular weight, amorphous engineering materials that have high impact strength over a wide temperature range. Polycarbonate resins are produced in U.S primarily by General Motors, the Bayer Corp and the Dow Chemical Co. Polycarbonates are generally unaffected by greases and oils but they are attached by most aromatic solvents, esters and ketones.

Nano Level:

Surface to volume ratio at micro level increases to 6000. However if the same ratio is studied on a nano scale, this ratio is 6,000,000,000. The reason behind this is that nano particles have small volume (mass) but large surface area. These nano particles are produced in large numbers. These particles behave like gases and disperse quickly. Nano particles agglomerate quickly after production. More and more, science materials are produced using nanotechnology. Both types of nanotechnology (passive and active) are used in the fabrication of products with their applications in chemicals, plastics and polymers, semiconductors and biomedical. Examples of nano products are quantum dots, carbon nano tubes, nano coatings, nano layers and nano shells. Nano scale materials exhibit new properties. Paints, pigments, inks, coatings, sunscreens and cosmetics are produced by nanotechnology. Modern research predicts that but (mass or volume) chemistry may be less important than surface area and surface chemistry for nano structured materials.

On the contrary, nano particles exposure is very dangerous. These particles deposit in the lungs and remain there for longer period of time. These particles interact with the body. Nano technology needs research to study work place conditions to cause exposure to nanoparticles. In a shocking discovery, nano particles are also found in the " nano-thermite dust" at World Trade Center , September 11 explosion. According to Danish Chemist "Dr. Niels Harrit, associate professor at the department of chemistry at Copenhagen University in Denmark, active thermatic material was discovered in dust from the "9-11 World Trade Center Catastrophe". Chemist says, "Nano-Thermite bombs brought down the WTC buildings. It was not the impact of planes that collapsed the three buildings at WTC but it was nano-thermite, an exotic compound, which melts metal virtually instantly. Its ignition temperature is far lower than that of conventional thermite. (Iron Oxide plus Aluminum). The properties of these red and gray chips were analyzed using optical microscopy, scanning electron microscopy (SEM), X-Ray energy dispersive spectroscopy (XED's) and differential scanning calorimetric (DSC). Chemists conclude that the red layers of the red and gray chips discovered in the WTC dust are active, unreacted thermatic material, incorporating Nanotechnology and is a highly energetic pyrotechnic or explosive material.

Objectives:

This unit is intended for students in grade ten, grade eleven and grades twelve. Students of tenth grade find several geometry connections in surface area and volume. They can spend five class periods of fifty minutes each to understand the surface area of planar and solid figures. Once they understand the concept of volume, they can establish the relation between surface area and volume. Students of grade eleven can connect to the rate of chemical reaction and speed of evaporation. However, students of twelfth grade find physics connection in this unit. They will be able to understand the concept of surface – tension, evaporation, surface area/volume ratio at macro, micro and nano level.

The objectives of the unit include the following:

- Use proportions to find measures in similar figures.
- Estimate the quantity/number if the linear relation is known.
- Determine the relationship between corresponding parts of similar triangles and apply this knowledge to solve problems.
- Establish a relation between surface area and volume.
- Predict the strength of an object, once the surface area to volume ratio is known.
- Explore information once surface tension of a liquid is known.
- Raise awareness in constructing designs and structures on the basis of surface area and volume ratio.

Strategies:

The above mentioned objectives can be achieved by using the following strategies:

a) Alternative teaching strategy
b) Interdisciplinary connection
c) Inclusion strategy
d) Enrichment
e) Co-operative learning
f) Differentiated Instructions

An effective "Interdisciplinary general education program regularly reviews its goals, its curriculum, and the courses which it offers. In addition, the program has a process for regularly monitoring progress toward achieving integrative goals and outcomes, curriculum and courses.

"Enrichment" is the most effective strategy for students to research on a problem. In my unit on surface to volume ratio, students are open to research on surface/volume ratio for different solids. For example, SVR for cube is 6/S. By assigning different values to "S" they conclude that by reducing the value of "S", SVR increases. Thus at nano level, SVR is quite big in comparison to micro level. The same kind of research can be performed for SVR for Spheres.

Differentiated instruction is another strategy I would use to plan my lesson, keeping in mind that each student has different needs at a certain time. This strategy involves all students in the lesson through the use of questioning aimed at different levels of thinking. Students are evaluated on their individual differences. Today's classrooms contain students representing genders and multiple cultures, ESL's and special Ed students with diverse IEP's. Differentiated instruction is most effective teaching strategy in these classrooms.

Classroom Activities:

Activities:
Each student will select one of the following projects for a culminating activity: Surface volume ratio is of critical importance to all living things. It explains which elephants have big ears, why hippos and rhinos have short, thick legs and must spend a lot of time in water and why movie monsters like King Kong and Godzilla cant exist.

Activity on Convenient Sizes: (Body Temperature): each living thing processes food for energy. This energy creates heat and radiates through its body surface. Imagine two similar animals, one with dimensions four times as large as those of the other. Students are able to answer the following questions after understanding of the terms: Surface area and volume, heat and temperature, evaporation and radiation.

Question: How would the surface areas of these animals compare? How much more heat could the larger animals radiate through the surface?
Answer: Surface area involves two dimensions, so the ratio of surface areas is 1:16. Thus larger animal can radiate heat 16 times more than the smaller animal.

Question: How do the volumes of these two animals compare? How many times (heat or energy) will be produced by larger animals as compared to the small animal?
Answer: Volume involves three dimensions, so the ratio of volume is 1:64. As heat (energy) produced is proportional to the animal's volume, so energy or heat generated by the larger animal will be sixty four times as large as the smaller ones.

Question: Why do large animals cool down their body temperature slowly when compared to the small animals?
Answer: Surface area of larger animals as discussed above is sixteen times more than that of the smaller animal. And the heat generated by the larger animal is sixty four times than that of the smaller one. Therefore, each square unit of surface of larger animal must radiate four times as much heat as the smaller animal. So the larger animals cool down slowly than the smaller ones.

Question: If a woman and a small child fall into a cold lake, why is the child in greater danger of hypothermia?
Answer: Because of the large surface area of woman, she will cool down slowly but the child will cool rather quickly. So the child is in danger of Hyperthermia.

Surface to Volume Ratio sometimes gives an idea of estimation also:

Example: There is a 17.68-meter tall statue of Bahubali, in Sravanabelagola, India. Worshipers of Jain religion bathe this statue with coconut milk in every 12 years. It is estimated that one average sized coconut is just enough to bathe .61-meter tall statue. Estimate the number of coconuts needed to bathe the original statue.

Solution: using surface ratio of similarity:

$$N1/N2 = S1/S2 = H1/H2 = N1/1 = (17.68)(17.68) / (0.61)(0.61) = N1 = 840$$

Thus, 840 coconuts are required to bathe Bahubali.

Example: A rectangular prism oil tank holds 81 cubic meter of furnace oil. A similar shaped oil tanks holds only 3 cubic meter of furnace oil. If a 1.2 square meter cover fits on the small tank, what is the area of cover that will fit on the larger tank?
Solution: $3\sqrt{v1/v2} = \sqrt{S1/S2} = L1/L2$
$3\sqrt{81/3} = \sqrt{S1/1.2}$
$3 = \sqrt{S1/1.2}$
$9 = S1/1.44$
$S1 = 12.96$ meter squared.

VI: Resources:

- http://www.sciencedaily.com/releases/2009/04/090408140223.htm
- *Informal Geometry* Prentice Hall (A Paramount Communication Company), *Geometry* (Applying, Reasoning and Measuring) McDougall Littell
- *Discovering Geometry* (An Investigative Approach) (Key Curriculum Press)
- *Introductory Chemistry* by John P Sevenat, American Free Press: 645, Pennsylvania Ave. SE, Suite 100, Washington, D.C 2003.
- www. americanfreepress.net.

Appendix/ Standards:

The core curriculum of the School District Of Philadelphia is aligned to Pennsylvania Academic standards for geometry, Mathematics, physics and Physical Science. These standards include instruction on the following topics: concepts of surface area, volume, surface tension, evaporation and elasticity.

2.3A: Geometry in nature and in art.

2.2A: Performs calculations requiring multiple procedures and operations.

2.2C: Area of rectangles and Parallelograms. Explain and apply the relationship between corresponding parts of similar triangles.

2.3 C: Determine reasonable estimates for multi step problems that involve conversions of units of measure.

2.5 B: Determine measurements using appropriate rounding techniques.

2.7 C: Explain and apply the relationship between the areas and volumes of similar figures.

2.9 G: Use appropriate geometric notation and terminology in communicating results and ideas.

2.11 A: Apply algebraic logic and skill s to solve problems involving circles and polygons.

3.4 C: Accelerating electric charges produce electromagnetic waves around them.